8.5 Answers

8.5.1 Solution to 8.3 Example

$$y = \sqrt{\sin x}$$

(i)

x	0	$\frac{\pi}{8}$	$\frac{2\pi}{8}$	$\frac{3\pi}{8}$	$\frac{4\pi}{8}$
y	0	0.619	p	0.961	q

$$p = \sqrt{\sin\left(\frac{2\pi}{8}\right)} = \sqrt{\frac{\sqrt{2}}{2}} = 0.841$$

$$q = \sqrt{\sin\left(\frac{4\pi}{8}\right)} = \sqrt{1} = 1.000$$

[2 marks]

(ii) With four strips the trapezium rule becomes,

$$\int_a^b y\,dx \approx \frac{1}{2}h\{\,(y_0 + y_4) + 2(y_1 + y_2 + y_3)\,\}$$

$$A = \int_0^{\frac{\pi}{2}} \sqrt{\sin x}\,dx \approx \frac{1}{2} \times \frac{\pi}{8}\{(0 + 1) + 2(0.619 + 0.841 + 0.961)\}$$

$$\approx \frac{\pi}{16}\{1 + 2 \times 2.421\}$$

$$\approx 1.147$$

[3 marks]

(iii)

$$B = \int_0^{\frac{\pi}{2}} \left(\sqrt{\sin x} + 3\right) dx$$

$$= \int_0^{\frac{\pi}{2}} \sqrt{\sin x}\,dx + \int_0^{\frac{\pi}{2}} 3\,dx$$

$$\approx 1.147 + \left[\,3x\,\right]_0^{\frac{\pi}{2}}$$

$$\approx 1.147 + 3 \times \frac{\pi}{2}$$

$$\approx 5.859$$

[2 marks]

8.5.2 Solutions to 8.4 Exercise

Answer 1

$$\sqrt{1 + \sin x}$$

(i)

x	0	0.5	1	1.5	2
y	1	1.216	p	1.413	q

$$p = \sqrt{1 + \sin(1)} = 1.357$$

$$q = \sqrt{1 + \sin(2)} = 1.382$$

[2 marks]

(ii) With four strips the trapezium rule becomes,

$$\int_a^b y\,dx \approx \frac{1}{2}h\{(y_0 + y_4) + 2(y_1 + y_2 + y_3)\}$$

$$A = \int_0^2 \sqrt{1 + \sin x}\;dx \approx \frac{0.5}{2}\{(1 + 1.382) + 2(1.216 + 1.357 + 1.413)\}$$

$$\approx 0.25\{2.382 + 2 \times 3.986\}$$

$$\approx 2.589$$

[3 marks]

(iii)

$$B = \int_0^2 x + \sqrt{1 + \sin x}\;dx$$

$$= \int_0^2 x\,dx + \int_0^2 \sqrt{1 + \sin x}\;dx$$

$$\approx \left[\frac{x^2}{2}\right]_0^2 + 2.589$$

$$\approx 2 + 2.589$$

$$\approx 4.589$$

[2 marks]

Answer 2

$$y = e^x\sqrt{\sin x}$$

(a)

x	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π
y	0	1.84432	4.81047	8.87207	0

[2 marks]

(b) With four strips the trapezium rule becomes,

$$\int_a^b y\,dx \approx \frac{1}{2}h\left\{ \left(y_0 + y_4\right) + 2\left(y_1 + y_2 + y_3\right) \right\}$$

$$R = \int_0^{\pi} e^x\sqrt{\sin x}\;dx$$

$$\approx \frac{1}{2} \times \frac{\pi}{4}\{(0 + 0) + 2(1.84432 + 4.81047 + 8.87207)\}$$

$$\approx \frac{\pi}{8}\{0 + 2 \times 15.52686\}$$

$$\approx 12.1948$$

[4 marks]

Answer 3

$$y = sec\left(\frac{1}{2}x\right)$$

(a)

x	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
y	1	1.035276	1.154701	1.414214

[1 mark]

(b) With three strips the trapezium rule becomes,

$$\int_a^b y\,dx \approx \frac{1}{2}h\left\{ \left(y_0 + y_3\right) + 2\left(y_1 + y_2\right) \right\}$$

$$\approx \frac{1}{2} \times \frac{\pi}{6}\{(1 + 1.414214) + 2(1.035276 + 1.154701)\}$$

$$\approx \frac{\pi}{12}\{2.414214 + 2 \times 2.189977\}$$

$$\approx 1.7787$$

[3 marks]

Answer 4

$$y = \frac{2\,sin\,2x}{(1 + cos\,x)}$$

(a)

x	0	$\frac{\pi}{8}$	$\frac{\pi}{4}$	$\frac{3\pi}{8}$	$\frac{\pi}{2}$
y	0	0.73508	1.17157	1.02280	0

[1 mark]

(b) With four strips the trapezium rule becomes,

$$\int_a^b y\,dx \approx \frac{1}{2}h\{\,(y_0 + y_4) + 2(y_1 + y_2 + y_3)\,\}$$

$$\approx \frac{1}{2} \times \frac{\pi}{8}\{(0 + 0) + 2\,(0.73508 + 1.17157 + 1.02280)\}$$

$$\approx \frac{\pi}{16}\{\,0 + 2 \times 2.92945\,\}$$

$$\approx 1.1504$$

[3 marks]

Answer 5

$$y = \sqrt{0.75 + cos^2 x}$$

(a)

x	0	$\frac{\pi}{12}$	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$
y	1.3229	1.2973	1.2247	1.1180	1

[2 marks]

(b) **(i)** With two strips the trapezium rule becomes,

$$\int_a^b y\,dx \approx \frac{1}{2}h\left\{ \left(y_0 + y_2\right) + 2\left(y_1\right) \right\}$$

$$\approx \frac{1}{2} \times \frac{\pi}{6}\{(1.3229 + 1) + 2\,(1.2247)\}$$

$$\approx \frac{\pi}{12}\{2.3229 + 2 \times 1{,}2247\}$$

$$\approx 1.249$$

[3 marks]

(ii) With four strips the trapezium rule becomes,

$$\int_a^b y\,dx \approx \frac{1}{2}h\left\{ \left(y_0 + y_4\right) + 2\left(y_1 + y_2 + y_3\right) \right\}$$

$$\approx \frac{1}{2} \times \frac{\pi}{12}\{(1.3229 + 1) + 2\,(1.2973 + 1.2247 + 1.1180)\}$$

$$\approx \frac{\pi}{24}\{2.3229 + 2 \times 3.6400\}$$

$$\approx 1.257$$

[3 marks]

Answer 6

(a) With four strips the trapezium rule becomes,

$$\int_a^b y\,dx \approx \frac{1}{2}h\left\{ \left(y_0 + y_4 \right) + 2\left(y_1 + y_2 + y_3 \right) \right\}$$

$$R \approx \frac{1}{2} \times \frac{1}{2}\left\{\left(\frac{11\sqrt{5}}{5} + \frac{13}{3}\right) + 2\left(\frac{23\sqrt{6}}{12} + \frac{12\sqrt{7}}{7} + \frac{25\sqrt{2}}{8}\right)\right\}$$

$$\approx \frac{1}{2} \times \frac{1}{2}\{(4.91 + 4.33 + 2(4.69 + 4.53 + 4.41)\}$$

$$\approx 9.14$$

[4 marks]

(b)

$$\int_4^6 \frac{x + 7}{\sqrt{2x - 3}}\,dx = \int_5^9 \frac{u + 3 + 14}{2\sqrt{u}} \; \frac{du}{2}$$

$$= \int_5^9 \frac{u + 17}{4\sqrt{u}}\,du$$

$$= \frac{1}{4}\int_5^9 u^{\frac{1}{2}} + 17u^{-\frac{1}{2}}\,du$$

$$= \frac{1}{4}\left[\frac{2u^{\frac{3}{2}}}{3} + 34u^{\frac{1}{2}}\right]_5^9$$

$$= 30 - \frac{28}{3}\sqrt{5}$$

[7 marks]