2.6 Answers

2.6.1 Solution to 2.3.2 Teaching Example #2

Without using a calculator, show that 504091 is divisible by 7

$$504091 \Rightarrow 50409 - 2 = 50407$$
$$\Rightarrow 5040 - 14 = 5026$$
$$\Rightarrow 502 - 12 = 490 \text{ which is obviously divisible by 7}$$
$$\therefore \text{ 504091 is divisible by 7}$$

[2 marks]

2.6.2 Solution to 2.3.3 Teaching Example #3

Provide a proof for the test for divisibility by 7

Observe that any natural number, $\mathbb{N}$, can be expressed in algebra as aR where R is the single rightmost digit, and a is all the other digits.
When aR is divisible by 7,

$$\begin{aligned} aR &= 10a + R && \text{is divisible by 7} \\ &= 3a + 7a + R && \text{is divisible by 7} \\ &= 3a + 7a + R - 7R && \text{is divisible by 7} \\ &= 3a - 6R && \text{is divisible by 7} \\ &= 3(a - 2R) && \text{is divisible by 7} \end{aligned}$$

As 3 is not divisible by 7, $(a - 2R)$ must be. ☐

[7 marks]

2.6.3 Solutions to 2.5 Exercise

Answer 1

(i) $24094 \Rightarrow 2409 - 8 = 2401$

$$\Rightarrow 240 - 2 = 238$$
$$\Rightarrow 23 - 16 = 7 \text{ which is obviously divisible by 7}$$
$$\therefore \text{ 24094 is divisible by 7}$$

[2 marks]

(ii) As 24094 ends in a 4, 24094 is an even number.
Thus, 24094 is divisible by both 2 and by 7
That is, 24094 is divisible by 14

[2 marks]

Answer 2

$91 \;\Rightarrow\; 9 - 2 = 7$ which is obviously divisible by 7

$\therefore$ 91 is divisible by 7

[1 mark]

Answer 3

$$\begin{aligned} abba &= 1000a + 100b + 10b + a \\ &= 1001a + 110b \\ &= 11(91a + 10b) \end{aligned}$$

which is divisible by 11 $\square$

[4 marks]

Answer 4

$$\begin{aligned} aba &= 100a + 10b + a \quad \Rightarrow \quad aba = 101a + 10b \\ bab &= 100b + 10a + b \quad \Rightarrow \quad bab = 10a + 101b \end{aligned}$$

$$\begin{aligned} aba + bab &= 111a + 111b \\ &= 37 \times 3(a + b) \end{aligned}$$

which is divisible by 37 $\square$

[5 marks]

Answer 5

(i)

$$\begin{aligned} abcd &= 1000a + 100b + 10c + d \\ &= 999a + a + 99b + b + 9c + c + d \\ &= (999a + 99b + 9c) + (a + b + c + d) \\ &= 3(333a + 33b + 3c) + (a + b + c + d) \end{aligned}$$

Clearly, $3(333a + 33b + 3c)$ is divisible by 3

By the Sum Divisor Rule, when $abcd$ and $3(333a + 33b + 3c)$ are both divisible by 3, so must $(a + b + c + d)$ be.

$\therefore$ $abcd$ is divisible by 3 when the sum of the digits is divisible by 3 $\square$

[5 marks]

(ii) 7458 is divisible by 3 because 7 + 4 + 5 + 8 = 24 which is divisible by 3

[2 marks]

Answer 6

(i) Observe that any natural number, $\mathbb{N}$, of more than 2 digits can be expressed in algebra as aR where R is the two rightmost digits, and a is all the other digits.

When aR is divisible by 4,

$$aR = 100a + R \qquad \text{is divisible by 4}$$

Clearly $100a$ is divisible by 4

By the Sum Divisor Rule, when aR and $100a$ are both divisible by 4, so must R be. In other words, and natural number of more than 2 digits is divisible by 4 precisely when the two rightmost digits are divisible by 4

[6 marks]

(ii) 290547452 is divisible by 4 because the two rightmost digits, 52, are divisible by 4. That is $52 = 4 \times 13$ (pack of cards).

[2 marks]

Answer 7

(i)

$$119483 \Rightarrow 11948 + 12 = 11960$$
$$\Rightarrow 1196 + 0 = 1196$$
$$\Rightarrow 119 + 24 = 143$$
$$\Rightarrow 14 + 12 = 26 \text{ which is obviously divisible by 13}$$
$$\therefore\ 119483 \text{ is divisible by 13}$$

[2 marks]

(ii)

$$504088 \Rightarrow 50408 + 32 = 50440$$
$$\Rightarrow 5044 + 0 = 5044$$
$$\Rightarrow 504 + 16 = 520 \text{ which is obviously divisible by 13}$$
$$\therefore\ 504088 \text{ is divisible by 13}$$

[2 marks]

(iii) Observe that any natural number, $\mathbb{N}$, can be expressed in algebra as aR where R is the single rightmost digit, and a is all the other digits.

When aR is divisible by 13,

$$aR = 10a + R \qquad \text{is divisible by 13}$$
$$= 10a - 13a + R - 13R \qquad \text{is divisible by 13}$$
$$= -3a - 12R \qquad \text{is divisible by 13}$$
$$= (-3)(a + 4R) \qquad \text{is divisible by 13}$$

As (-3) is not divisible by 13, $(a + 4R)$ must be. ☐

[7 marks]