Lesson 7

A-Level Pure Mathematics, Year 2

Proof I : The Art of Absolute Certainty

7.1 Test

Any solution based entirely on graphical or numerical methods is not acceptable

Marks Available : 50 marks

Question 1

Find a counterexample that disproves the statement, “All numbers which are one greater than a multiple of 24 are the squares of prime numbers”

[2 marks]

Question 2

Prove than the sum of the squares of any two consecutive integers is an odd number.

[4 marks]

Question 3

Write down any 2-digit number *ab* then repeat the digits twice to form the 6-digit number *ababab*

Explain why the 6-digit number will be divisible by 3, by 7, by 13 and by 37

[4 marks]

Question 4

Prove that the product of two consecutive even numbers is divisible by 8

[6 marks]

Question 5

The n^{th} triangular number is of the form, $T_n = \dfrac{n(n+1)}{2}$

(i) Calculate $T_7 + T_8$

[2 marks]

(ii) Write down the formula for the $(n+1)^{\text{th}}$ triangular number

[2 marks]

(iii) Prove that the sum of any two consecutive triangular numbers is a square number

[4 marks]

Question 6

Divisibility by 11

Subtract the last digit from the number formed by the remaining digits.

Repeat as necessary.

If, at any stage, a result is obtained that is obviously divisible by 11 then the original number is also divisible by 11

(i) Without using a calculator, show that 3531 is divisible by 11

[2 marks]

(ii) Without using a calculator, show that 1984422 is divisible by 11

[2 marks]

(iii) Provide a proof that explains why the test for divisibility by 11 works

[7 marks]

Question 7

(i) Prove by contradiction that if n^3 is even, then n is even.

[3 marks]

(ii) Hence prove by contradiction that $\sqrt[3]{2}$ is an irrational number.

[6 marks]

Question 8

Consider the following sequence of calculations,

$$x_1 = 1^2 + 2^2 + (1 \times 2)^2$$
$$x_2 = 2^2 + 3^2 + (2 \times 3)^2$$
$$x_3 = 3^2 + 4^2 + (3 \times 4)^2$$
$$x_4 = 4^2 + 5^2 + (4 \times 5)^2$$
$$\ldots$$

Show that x_n will always be a perfect square.

[6 marks]

This document is a part of a **Mathematics Community Outreach Project** initiated by Shrewsbury School
It may be freely duplicated and distributed, unaltered, for non-profit educational use
In October 2020, Shrewsbury School was voted “**Independent School of the Year 2020**”
© 2026 Number Wonder
Teachers may obtain detailed worked solutions to the exercises by email from MHHShrewsbury@Gmail.com