4.6 Answers to 4.5 Exercise

Answer 1

(i) $\dfrac{d^2y}{dx^2} - 3\dfrac{dy}{dx} - 28y = 0$

$m^2 - 3m - 28 = 0$ the auxiliary equation

$(m + 4)(m - 7) = 0$

Either $m = -4$ or $m = 7$

$y = Ae^{-4x} + Be^{7x}$ for any arbitrary real constants A and B

[3 marks]

(ii) $16\dfrac{d^2y}{dx^2} + 8\dfrac{dy}{dx} + y = 0$

$16m^2 + 8m + 1 = 0$ the auxiliary equation

$(4m + 1)(4m + 1) = 0$

$m = -\dfrac{1}{4}$

$y = (A + Bx)\, e^{-\frac{1}{4}x}$

for any arbitrary real constants A and B

[3 marks]

Answer 2

$\dfrac{d^2y}{dx^2} + 8\dfrac{dy}{dx} + 25y = 0$

$m^2 + 8m + 25 = 0$ the auxiliary equation

$m = \dfrac{-8 \pm \sqrt{64 - 4 \times 1 \times 25}}{2}$

$= -4 \pm 3\mathrm{i}$

$y = e^{-4x}(A\cos(3x) + B\sin(3x))$

for any arbitrary real constants A and B

[3 marks]

Answer 3

(i) $y'' + 4y' + 5y = 0$

$$m^2 + 4m + 5 = 0$$

$$m = \frac{-4 \pm \sqrt{16 - 4 \times 1 \times 5}}{2}$$

$$= -2 \pm \mathrm{i}$$

$$y = e^{-2x}(A\cos(x) + B\sin(x))$$

for any arbitrary real constants A and B

[3 marks]

(ii) As $y(0) = 0$,

$$0 = e^0(A\cos(0) + B\sin(0))$$

$$0 = A$$

$$\therefore \; y = e^{-2x}(B\sin(x))$$

$$\frac{dy}{dx} = Be^{-2x}\cos(x) - 2Be^{-2x}\sin(x)$$

As $y'(0) = 2$,

$$2 = Be^0\cos(0) - 2Be^0\sin(0)$$

$$2 = B$$

The particular solution is thus,

$$y = 2e^{-2x}\sin(x)$$

[5 marks]

Answer 4

(i) $15y'' - 7y' - 2y = 0$

$15m^2 - 7m - 2y = 0$ the auxiliary equation

$(5m + 1)(3m - 2) = 0$

Either $m = -\frac{1}{5}$ or $m = \frac{2}{3}$

$y = Ae^{-\frac{1}{5}x} + Be^{\frac{2}{3}x}$ for any arbitrary real constants A and B

[3 marks]

(ii) $4y'' + 20y' + 25y = 0$

$4m^2 + 20m + 25 = 0$ the auxiliary equation

$(2m + 5)(2m + 5) = 0$

$m = -\frac{5}{2}$

$y = (A + Bx)\, e^{-\frac{5}{2}x}$

for any arbitrary real constants A and B

[3 marks]

(iii) $y'' + \sqrt{3}\, y' + 3y = 0$

$m^2 + \sqrt{3}\, m + 3 = 0$ the auxiliary equation

$$m = \frac{-\sqrt{3} \pm \sqrt{3 - 4 \times 1 \times 3}}{2}$$

$$= -\frac{\sqrt{3}}{2} \pm \frac{3}{2}\mathrm{i}$$

$$y = e^{-\frac{\sqrt{3}}{2}x}\left(A\cos\left(\frac{3x}{2}\right) + B\sin\left(\frac{3x}{2}\right)\right)$$

for any arbitrary real constants A and B

[3 marks]

Answer 5

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 10y = 0$$

$$m^2 - 2m + 10 = 0 \quad \text{the auxiliary equation}$$

$$m = \frac{2 \pm \sqrt{4 - 4 \times 1 \times 10}}{2}$$

$$= 1 \pm 3\text{i}$$

$$y = e^x \left(A\cos(3x) + B\sin(3x) \right)$$

for any arbitrary real constants A and B

As $y = 0$ when $x = 0$,

$$0 = e^0 \left(A\cos(0) + B\sin(0) \right)$$

$$0 = A$$

$$\therefore \quad y = e^x (B\sin(3x))$$

As $\frac{dy}{dx} = 3$ when $x = 0$,

$$\frac{dy}{dx} = 3B\,e^x \cos(3x) + B\,e^x \sin(3x)$$

$$3 = 3B\,e^0 \cos(0) + 2\,e^0 \sin(0)$$

$$3 = 3B$$

$$B = 1$$

The particular solution is thus,

$$y = e^x \sin(3x)$$

[8 marks]