5.4 Answers

5.4.1 Solutions to 5.2 Example

$$U_1 = \frac{1}{3}, \quad U_{n+1} = \frac{2 - U_n}{2}$$

$$U_2 = \frac{\left(2 - \frac{1}{3}\right)}{2} \times \frac{3}{3} = \frac{6 - 1}{6} = \frac{5}{6}$$

$$U_3 = \frac{\left(2 - \frac{5}{6}\right)}{2} \times \frac{6}{6} = \frac{12 - 5}{12} = \frac{7}{12}$$

$$U_4 = \frac{\left(2 - \frac{7}{12}\right)}{2} \times \frac{12}{12} = \frac{24 - 7}{24} = \frac{17}{24}$$

$$U_5 = \frac{\left(2 - \frac{17}{24}\right)}{2} \times \frac{24}{24} = \frac{48 - 17}{48} = \frac{31}{48}$$

$$U_6 = \frac{\left(2 - \frac{31}{48}\right)}{2} \times \frac{48}{48} = \frac{96 - 31}{96} = \frac{65}{96}$$

U_1	U_2	U_3	U_4	U_5	U_6
$\frac{1}{3}$	$\frac{5}{6}$	$\frac{7}{12}$	$\frac{17}{24}$	$\frac{31}{48}$	$\frac{65}{96}$

[6 marks]

5.4.2 Solutions to 5.3 Exercise

Answer 1

(i) $14 \times \frac{5}{7} = 10$

$= 2 \times 5$

$= 10$

(ii) $5\left(3 + \frac{1}{5}\right)$

$= 15 + 1$

$= 16$

(iii) $\left(\frac{5}{7} + 3\right) \times 7$

$= 5 + 21$

$= 26$

(iv) $\left(7 + \frac{3}{4}\right) \times 4$

$= 28 + 3$

$= 31$

[4 marks]

Answer 2

(i) $\dfrac{\left(1 + \frac{1}{2}\right)}{13} \times \dfrac{2}{2} = \dfrac{2 + 1}{26} = \dfrac{3}{26}$

[2 marks]

(ii) $\dfrac{\left(3 + \frac{1}{4}\right)}{7} \times \dfrac{4}{4} = \dfrac{12 + 1}{28} = \dfrac{13}{28}$

[2 marks]

(iii) $\dfrac{\left(\frac{5}{7} + 2\right)}{6} \times \dfrac{7}{7} = \dfrac{5 + 14}{42} = \dfrac{19}{42}$

[2 marks]

(iv) $\dfrac{\left(11 + \frac{5}{9}\right)}{7} \times \dfrac{9}{9} = \dfrac{99 + 5}{63} = \dfrac{104}{63}$

[2 marks]

(v) $\dfrac{\left(4 + \frac{3}{8}\right)}{8} \times \dfrac{8}{8} = \dfrac{32 + 3}{64} = \dfrac{35}{64}$

[2 marks]

(vi) $\dfrac{\left(\frac{8}{3} + 7\right)}{4} \times \dfrac{3}{3} = \dfrac{8 + 21}{12} = \dfrac{29}{12}$

[2 marks]

Answer 3

(i) $\dfrac{\left(3 - \frac{1}{2}\right)}{2} \times \dfrac{2}{2} = \dfrac{6 - 1}{4} = \dfrac{5}{4}$

(ii) $A_1 = \dfrac{1}{2}$, $A_{n+1} = \dfrac{3 - A_n}{2}$

$A_2 = \dfrac{5}{4}$ (from part (i))

$A_3 = \dfrac{\left(3 - \frac{5}{4}\right)}{2} \times \dfrac{4}{4} = \dfrac{12 - 5}{8} = \dfrac{7}{8}$

$A_4 = \dfrac{\left(3 - \frac{7}{8}\right)}{2} \times \dfrac{8}{8} = \dfrac{24 - 7}{16} = \dfrac{17}{16}$

$A_5 = \dfrac{\left(3 - \frac{17}{16}\right)}{2} \times \dfrac{16}{16} = \dfrac{48 - 17}{32} = \dfrac{31}{32}$

$A_6 = \dfrac{\left(3 - \frac{31}{32}\right)}{2} \times \dfrac{32}{32} = \dfrac{96 - 31}{64} = \dfrac{65}{64}$

A_1	A_2	A_3	A_4	A_5	A_6
$\frac{1}{2}$	$\frac{5}{4}$	$\frac{7}{8}$	$\frac{17}{16}$	$\frac{31}{32}$	$\frac{65}{64}$

[7 marks]

(iii) The denominators are powers of 2

The numerators are alternatively one more or one less than the denominator

$A_7 = \dfrac{127}{128}$ $A_8 = \dfrac{257}{256}$ $A_9 = \dfrac{511}{512}$

[2 marks]

Answer 4

(i) $\dfrac{\left(1 - \frac{3}{10}\right)}{2} \times \dfrac{10}{10} = \dfrac{10 - 3}{20} = \dfrac{7}{20}$

(ii) $B_1 = \dfrac{3}{10}$, $B_{n+1} = \dfrac{1 - B_n}{2}$

$B_2 = \dfrac{7}{20}$ (from part (i))

$B_3 = \dfrac{\left(1 - \frac{7}{20}\right)}{2} \times \dfrac{20}{20} = \dfrac{20 - 7}{40} = \dfrac{13}{40}$

$B_4 = \dfrac{\left(1 - \frac{13}{40}\right)}{2} \times \dfrac{40}{40} = \dfrac{40 - 13}{80} = \dfrac{27}{80}$

$B_5 = \dfrac{\left(1 - \frac{27}{80}\right)}{2} \times \dfrac{80}{80} = \dfrac{80 - 27}{160} = \dfrac{53}{160}$

$B_6 = \dfrac{\left(1 - \frac{53}{160}\right)}{2} \times \dfrac{160}{160} = \dfrac{160 - 53}{320} = \dfrac{107}{320}$

B_1	B_2	B_3	B_4	B_5	B_6
$\frac{3}{10}$	$\frac{7}{20}$	$\frac{13}{40}$	$\frac{27}{80}$	$\frac{53}{160}$	$\frac{107}{320}$

[7 marks]

(iii) Each denominators is double the previous denominator

Each numerator is previous denominator minus previous numerator

$B_7 = \dfrac{213}{640}$ $B_8 = \dfrac{427}{1280}$ $B_9 = \dfrac{853}{1560}$

[2 marks]

Answer 5

(i) $\dfrac{\left(1-\frac{2}{3}\right)}{3} \times \dfrac{3}{3} = \dfrac{3-2}{9} = \dfrac{1}{9}$

(ii) $Z_1 = \dfrac{2}{3}$, $Z_{n+1} = \dfrac{1-Z_n}{3}$

$Z_2 = \dfrac{1}{9}$ (from part (i))

$Z_3 = \dfrac{\left(1-\frac{1}{9}\right)}{3} \times \dfrac{9}{9} = \dfrac{9-1}{27} = \dfrac{8}{27}$

$Z_4 = \dfrac{\left(1-\frac{8}{27}\right)}{3} \times \dfrac{27}{27} = \dfrac{27-8}{81} = \dfrac{19}{81}$

$Z_5 = \dfrac{\left(1-\frac{19}{81}\right)}{3} \times \dfrac{81}{81} = \dfrac{81-19}{243} = \dfrac{62}{243}$

$Z_6 = \dfrac{\left(1-\frac{62}{243}\right)}{3} \times \dfrac{243}{243} = \dfrac{243-62}{729} = \dfrac{181}{729}$

Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
$\frac{2}{3}$	$\frac{1}{9}$	$\frac{8}{27}$	$\frac{19}{81}$	$\frac{62}{243}$	$\frac{181}{729}$

[7 marks]

(iii) Each denominators is triple the previous denominator

Each numerator is previous denominator minus previous numerator

$Z_7 = \dfrac{548}{2187}$ $Z_8 = \dfrac{1639}{6561}$ $Z_9 = \dfrac{4922}{19683}$

[2 marks]

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