

6.5 Answers

GCSE Mathematics Iteration

6.5.1 Solution to 5.2 Example

$$\frac{1}{\left(\frac{3}{4} + 1\right)} \times \frac{4}{4} = \frac{4}{3 + 4} = \frac{4}{7}$$

[2 marks]

6.5.2 Solution to 5.3 Example

$$\begin{aligned} \text{When } n = 1 \Rightarrow F_2 &= \frac{2}{F_1 + 1} = \frac{2}{3 + 1} \\ &= \frac{1}{2} \end{aligned}$$

$$\begin{aligned} \text{When } n = 2 \Rightarrow F_3 &= \frac{2}{F_2 + 1} = \frac{2}{\left(\frac{1}{2} + 1\right)} \times \frac{2}{2} \\ &= \frac{4}{1 + 2} \\ &= \frac{4}{3} \end{aligned}$$

$$\begin{aligned} \text{When } n = 3 \Rightarrow F_4 &= \frac{2}{F_3 + 1} = \frac{2}{\left(\frac{4}{3} + 1\right)} \times \frac{3}{3} \\ &= \frac{6}{4 + 3} \\ &= \frac{6}{7} \end{aligned}$$

$$\begin{aligned} \text{When } n = 4 \Rightarrow F_5 &= \frac{2}{F_4 + 1} = \frac{2}{\left(\frac{6}{7} + 1\right)} \times \frac{7}{7} \\ &= \frac{14}{6 + 7} \\ &= \frac{14}{13} \end{aligned}$$

$$\begin{aligned} \text{When } n = 5 \Rightarrow F_6 &= \frac{2}{F_5 + 1} = \frac{2}{\left(\frac{14}{13} + 1\right)} \times \frac{13}{13} \\ &= \frac{26}{14 + 13} \\ &= \frac{26}{27} \end{aligned}$$

F_1	F_2	F_3	F_4	F_5	F_6
$\frac{1}{1}$	$\frac{1}{2}$	$\frac{4}{3}$	$\frac{6}{7}$	$\frac{14}{13}$	$\frac{26}{27}$

[7 marks]

6.5.3 Solutions to Exercise 6.4

Answer 1

$$(i) \quad \frac{1}{\left(\frac{2}{3} + 5\right)} \times \frac{3}{3} = \frac{3}{2 + 15} = \frac{3}{17}$$

[2 marks]

$$(ii) \quad \frac{4}{\left(\frac{5}{2} + 3\right)} \times \frac{2}{2} = \frac{8}{5 + 6} = \frac{8}{11}$$

[2 marks]

$$(iii) \quad \frac{3}{\left(\frac{9}{4} - 1\right)} \times \frac{4}{4} = \frac{12}{9 - 4} = \frac{12}{5}$$

[2 marks]

$$(iv) \quad \frac{4}{\left(\frac{8}{13} + 3\right)} \times \frac{13}{13} = \frac{52}{8 + 39} = \frac{52}{47}$$

[2 marks]

Answer 2

$$(i) \quad \frac{1}{\left(\frac{4}{3} + 2\right)} \times \frac{3}{3} = \frac{3}{4 + 6} = \frac{3}{10}$$

[2 marks]

$$(ii) \quad \frac{5}{\left(\frac{3}{7} + 2\right)} \times \frac{7}{7} = \frac{35}{3 + 14} = \frac{35}{17}$$

[2 marks]

$$(iii) \quad \frac{4}{\left(\frac{23}{20} + 5\right)} \times \frac{20}{20} = \frac{80}{23 + 100} = \frac{80}{123}$$

[2 marks]

$$(iv) \quad \frac{7}{\left(\frac{100}{101} + 1\right)} \times \frac{101}{101} = \frac{707}{100 + 101} = \frac{707}{201}$$

[2 marks]

Answer 3

$$(i) \quad \frac{1}{\left(\frac{1}{2} + 1\right)} \times \frac{2}{2} = \frac{2}{1+2} = \frac{2}{3}$$

[2 marks]

$$(ii) \quad A_1 = \frac{1}{2}, \quad A_{n+1} = \frac{1}{A_n + 1}$$

$$A_2 = \frac{2}{3} \text{ (from part (i))}$$

$$A_3 = \frac{1}{\left(\frac{2}{3} + 1\right)} \times \frac{3}{3} = \frac{3}{2+3} = \frac{3}{5}$$

$$A_4 = \frac{1}{\left(\frac{3}{5} + 1\right)} \times \frac{5}{5} = \frac{5}{3+5} = \frac{5}{8}$$

$$A_5 = \frac{1}{\left(\frac{5}{8} + 1\right)} \times \frac{8}{8} = \frac{8}{5+8} = \frac{8}{13}$$

$$A_6 = \frac{1}{\left(\frac{8}{13} + 1\right)} \times \frac{13}{13} = \frac{13}{8+13} = \frac{13}{21}$$

A_1	A_2	A_3	A_4	A_5	A_6
$\frac{1}{2}$	$\frac{2}{3}$	$\frac{3}{5}$	$\frac{5}{8}$	$\frac{8}{13}$	$\frac{13}{21}$

[7 marks]**(iii)** It's all to do with the Fibonacci numbers !

$$A_7 = \frac{21}{34} \quad A_8 = \frac{34}{55} \quad A_9 = \frac{55}{89}$$

[2 marks]

Answer 4

$$(i) \quad \frac{1}{\left(\frac{2}{3} + 1\right)} \times \frac{3}{3} = \frac{3}{2+3} = \frac{3}{5}$$

[2 marks]

$$(ii) \quad B_1 = \frac{1}{3}, \quad B_{n+1} = \frac{1}{2B_n + 1}$$

$$B_2 = \frac{3}{5} \text{ (from part (i))}$$

$$B_3 = \frac{1}{\left(\frac{6}{5} + 1\right)} \times \frac{5}{5} = \frac{5}{6+5} = \frac{5}{11}$$

$$B_4 = \frac{1}{\left(\frac{10}{11} + 1\right)} \times \frac{11}{11} = \frac{11}{10+11} = \frac{11}{21}$$

$$B_5 = \frac{1}{\left(\frac{22}{21} + 1\right)} \times \frac{21}{21} = \frac{21}{22+21} = \frac{21}{43}$$

$$B_6 = \frac{1}{\left(\frac{42}{43} + 1\right)} \times \frac{43}{43} = \frac{43}{42+43} = \frac{43}{85}$$

B_1	B_2	B_3	B_4	B_5	B_6
$\frac{1}{3}$	$\frac{3}{5}$	$\frac{5}{11}$	$\frac{11}{21}$	$\frac{21}{43}$	$\frac{43}{85}$

[7 marks]

$$(iii) \quad \text{Let } B_n = \frac{P_n}{Q_n} \text{ in which case } B_{n+1} = \begin{cases} \frac{Q_n}{2Q_n+1} & \text{if } n \text{ is even} \\ \frac{Q_n}{2Q_n-1} & \text{if } n \text{ is odd} \end{cases}$$

$$B_7 = \frac{85}{171} \quad B_8 = \frac{171}{341} \quad B_9 = \frac{341}{683}$$

[2 marks]

Answer 5

$$(i) \quad \frac{1}{\left(\frac{3}{5} + 2\right)} \times \frac{5}{5} = \frac{5}{3 + 10} = \frac{5}{13}$$

[2 marks]

$$(ii) \quad Z_1 = \frac{1}{5}, \quad Z_{n+1} = \frac{1}{3Z_n + 2}$$

$$Z_2 = \frac{5}{13} \text{ (from part (i))}$$

$$Z_3 = \frac{1}{\left(\frac{15}{13} + 2\right)} \times \frac{13}{13} = \frac{13}{15 + 26} = \frac{13}{41}$$

$$Z_4 = \frac{1}{\left(\frac{39}{41} + 2\right)} \times \frac{41}{41} = \frac{41}{39 + 82} = \frac{41}{121}$$

$$Z_5 = \frac{1}{\left(\frac{123}{121} + 2\right)} \times \frac{121}{121} = \frac{121}{123 + 242} = \frac{121}{365}$$

$$Z_6 = \frac{1}{\left(\frac{363}{365} + 2\right)} \times \frac{365}{365} = \frac{365}{363 + 730} = \frac{365}{1093}$$

Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
$\frac{1}{5}$	$\frac{5}{13}$	$\frac{13}{41}$	$\frac{41}{121}$	$\frac{121}{365}$	$\frac{365}{1093}$

[7 marks]

$$(iii) \quad \text{Let } Z_n = \frac{P_n}{Q_n} \text{ in which case } Z_{n+1} = \begin{cases} \frac{Q_n}{3Q_n + 2} & \text{if } n \text{ is even} \\ \frac{Q_n}{3Q_n - 2} & \text{if } n \text{ is odd} \end{cases}$$

$$Z_7 = \frac{1093}{3119} \quad Z_8 = \frac{3119}{9355} \quad Z_9 = \frac{9355}{28067}$$

[2 marks]