8.2 Answers to 8.1 Homework (Revision)

GCSE Mathematics
Iteration

Answer 1

$$U_1 = 124, \quad U_{n+1} = \frac{U_n + 5}{3}$$

U_1	U_2	U_3	U_4	U_5	U_6
124	43	16	7	4	3

[7 marks]

Answer 2

(i) $4 \times \left(\frac{3}{4} + 1 \right)$

$= 3 + 4$

$= 7$

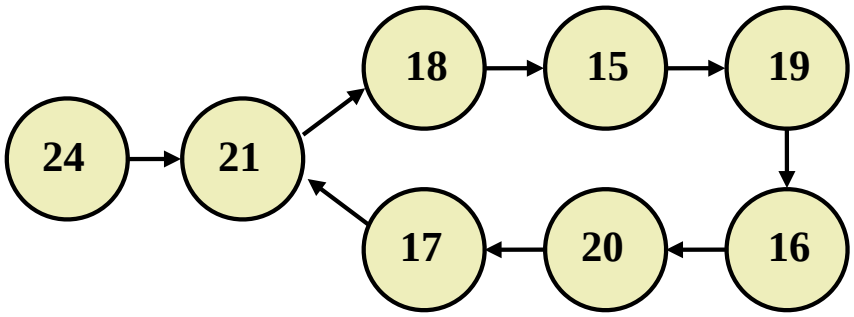
(ii) $\left(5 + \frac{3}{7} \right) \times 7$

$= 35 + 3$

$= 38$

[4 marks]

Answer 3



[8 marks]

Answer 4

(i) $\left(4 + \frac{1}{6} \right) \times 6 = 24 + 1 = 25$

[2 marks]

(ii) $\frac{\left(3 + \frac{2}{7} \right)}{4} \times \frac{7}{7} = \frac{21 + 2}{28} = \frac{23}{28}$

[2 marks]

Answer 5

$A_1 = \frac{1}{16} \qquad A_{n+1} = \frac{4}{3} A_n$

$A_2 = \frac{4}{3} \times \frac{1}{16} = \frac{1}{12}$

$A_3 = \frac{4}{3} \times \frac{1}{12} = \frac{1}{9}$

$A_4 = \frac{4}{3} \times \frac{1}{9} = \frac{4}{27}$

$A_5 = \frac{4}{3} \times \frac{4}{27} = \frac{16}{81}$

$A_6 = \frac{4}{3} \times \frac{16}{81} = \frac{64}{243}$

A_1	A_2	A_3	A_4	A_5	A_6
$\frac{1}{16}$	$\frac{1}{12}$	$\frac{1}{9}$	$\frac{4}{27}$	$\frac{16}{81}$	$\frac{64}{243}$

[7 marks]

Answer 6

(i) $\frac{\left(\frac{3}{8} + 2\right)}{5} \times \frac{8}{8} = \frac{3 + 16}{40} = \frac{19}{40}$

[2 marks]

(ii) $\frac{\left(5 + \frac{7}{4}\right)}{11} \times \frac{4}{4} = \frac{20 + 7}{44} = \frac{27}{44}$

[2 marks]

(iii) $\frac{\left(4 + \frac{2}{3}\right)}{5} \times \frac{3}{3} = \frac{12 + 2}{15} = \frac{14}{15}$

[2 marks]

Answer 7

(i) $\dfrac{\left(1 - \frac{1}{2}\right)}{3} \times \dfrac{2}{2} = \dfrac{2 - 1}{6} = \dfrac{1}{6}$

[2 marks]

(ii) $B_1 = \dfrac{1}{2}$, $B_{n+1} = \dfrac{1 - B_n}{3}$

$B_2 = \dfrac{1}{6}$ (from part (i))

$B_3 = \dfrac{\left(1 - \frac{1}{6}\right)}{3} \times \dfrac{6}{6} = \dfrac{6 - 1}{18} = \dfrac{5}{18}$

$B_4 = \dfrac{\left(1 - \frac{5}{18}\right)}{3} \times \dfrac{18}{18} = \dfrac{18 - 5}{54} = \dfrac{13}{54}$

$B_5 = \dfrac{\left(1 - \frac{13}{54}\right)}{3} \times \dfrac{54}{54} = \dfrac{54 - 13}{162} = \dfrac{41}{162}$

$B_6 = \dfrac{\left(1 - \frac{41}{162}\right)}{3} \times \dfrac{162}{162} = \dfrac{162 - 41}{486} = \dfrac{121}{486}$

B_1	B_2	B_3	B_4	B_5	B_6
$\frac{1}{2}$	$\frac{1}{6}$	$\frac{5}{18}$	$\frac{13}{54}$	$\frac{41}{162}$	$\frac{121}{486}$

[7 marks]

Answer 8

(i) $\dfrac{1}{\left(\frac{5}{3} + 4\right)} \times \dfrac{3}{3} = \dfrac{3}{5 + 12} = \dfrac{3}{17}$

[2 marks]

(ii) $\dfrac{5}{\left(\frac{7}{8} + 2\right)} \times \dfrac{8}{8} = \dfrac{40}{7 + 16} = \dfrac{40}{23}$

[2 marks]

(iii) $\dfrac{9}{\left(\frac{7}{5} - 1\right)} \times \dfrac{5}{5} = \dfrac{45}{7 - 5} = \dfrac{45}{2}$

[2 marks]

Answer 9

(i) $\dfrac{2}{\left(3 + \frac{2}{3}\right)} \times \dfrac{3}{3} = \dfrac{6}{9 + 2} = \dfrac{6}{11}$

[2 marks]

(ii) $Z_1 = 0,\ Z_{n+1} = \dfrac{2}{3 + Z_n}$

$Z_2 = \dfrac{2}{3 + 0} = \dfrac{2}{3}$

$Z_3 = \dfrac{6}{11}$ (from part (i))

$Z_4 = \dfrac{2}{\left(3 + \frac{6}{11}\right)} \times \dfrac{11}{11} = \dfrac{22}{33 + 6} = \dfrac{22}{39}$

$Z_5 = \dfrac{2}{\left(3 + \frac{22}{39}\right)} \times \dfrac{39}{39} = \dfrac{78}{117 + 22} = \dfrac{78}{139}$

$Z_6 = \dfrac{2}{\left(3 + \frac{78}{139}\right)} \times \dfrac{139}{139} = \dfrac{278}{417 + 78} = \dfrac{278}{495}$

Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
0	$\frac{2}{3}$	$\frac{6}{11}$	$\frac{22}{39}$	$\frac{78}{139}$	$\frac{278}{495}$

[7 marks]