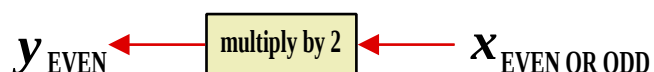
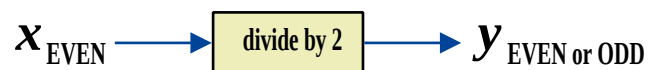


2.5 Answers

A-Level Mathematics Project The Collatz Conjecture

2.5.1 Solutions to 2.2 Grow From The Base

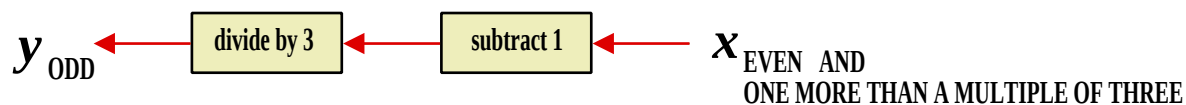
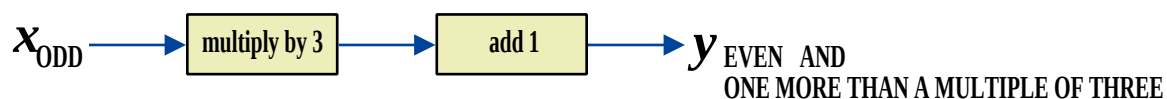
Reversing the rule for when x is even...



$$y_{REVERSE} = 2x$$

[2 marks]

Reversing the rule for when x is odd...



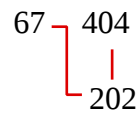
$$y_{REVERSE} = \frac{x - 1}{3}$$

[3 marks]

2.5.2 Solutions to 2.3 Exercise

Answer 1

(i) Reversing 202



The always occurring reverse is $y_{REVERSE} = 2x = 2 \times 202 = 404$

As 202 is even so ask yourself, is it also one more than a multiple of three ?

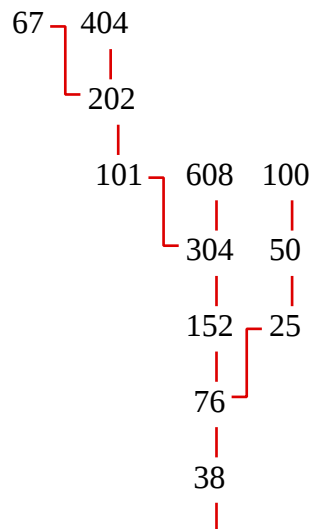
It is because 202 divided by 3 has remainder one third (remainder 0.33333...)

There is a second reversal $y_{REVERSE} = \frac{x-1}{3} = \frac{202-1}{3} = \frac{201}{3} = 67$

(This shades 67 in the *Collatz 100 Table*)

[2 marks]

(ii)



[3 marks]

Answer 2

(i) Reversing 42

$$\begin{array}{c} 84 \\ | \\ 42 \end{array}$$

The always occurring reverse is $y_{REVERSE} = 2x = 2 \times 42 = 84$

As 42 is even so ask yourself, is it also one more than a multiple of three ?

It is **NOT**: 42 divided by 3 has remainder zero thirds.

There is no second reverse.

(This shades 84 in the *Collatz 100 Table*)

[2 marks]

(ii) Reversing 256

$$\begin{array}{cc} 85 & 512 \\ & | \\ & 256 \end{array}$$

The always occurring reverse is $y_{REVERSE} = 2x = 2 \times 256 = 512$

As 256 is even so ask yourself, is it also one more than a multiple of three ?

It is because 256 divided by 3 has remainder one third (remainder 0.33333...)

There is a second reversal $y_{REVERSE} = \frac{x-1}{3} = \frac{256-1}{3} = \frac{255}{3} = 85$

(This shades 85 in the *Collatz 100 Table*)

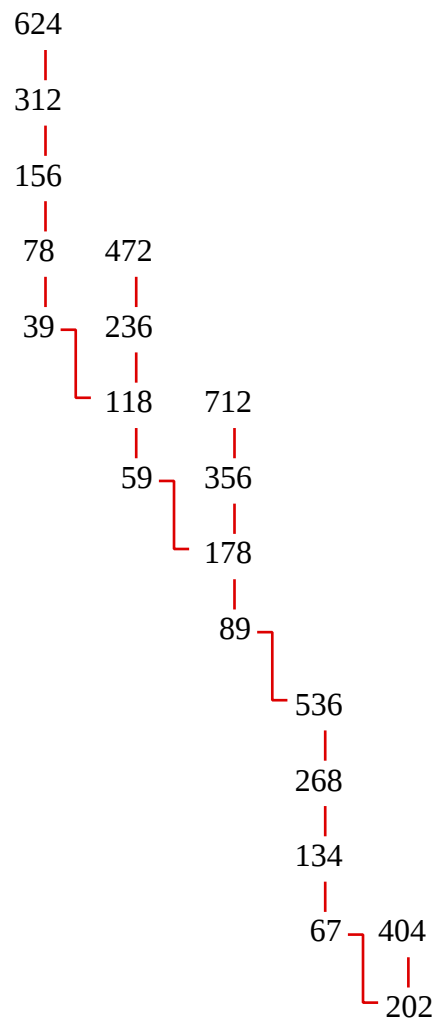
[2 marks]

(iii)

$$\begin{array}{ccccc} 84 & 85 & & 512 & \\ | & & & | & \\ 42 & & & 256 & \\ | & & & | & \\ 21 & & & 128 & \\ & & & | & \\ & & & 64 & \\ & & & | & \\ & & & 32 & \end{array}$$

[3 marks]

Answer 3



(This shades 39, 59, 78, and 89 in the *Collatz 100 Table*)

The updated *Collatz 100 Table* is;

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

[10 marks]

Answer 4

Here is a short program in Python

(Written by “Dawg” in 2015 on the stackoverflow website) :

```
print("This is The Collatz Sequence")
user = int(input("Enter a number: "))
def collatz(n):
    print(n)
    while n != 1:
        if n % 2 == 0:
            n = n // 2
            yield(n)
        else:
            n = n * 3 + 1
            yield(n)
print(list(collatz(user)))
```

Sample of output:

```
This is The Collatz Sequence
Enter a number: 3
3
[10, 5, 16, 8, 4, 2, 1]
```

[8 bonus marks]