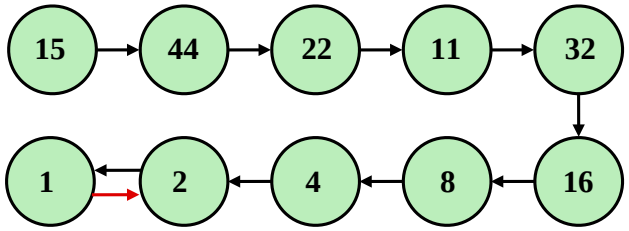
4.4 Answers

4.4.1 Solutions to 4.2 If At First You Don't Succeed

(i)

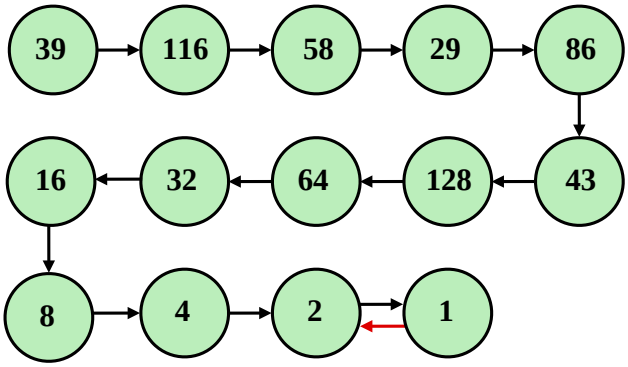


[4 marks]

4.4.2 Solution to 4.3 Exercise

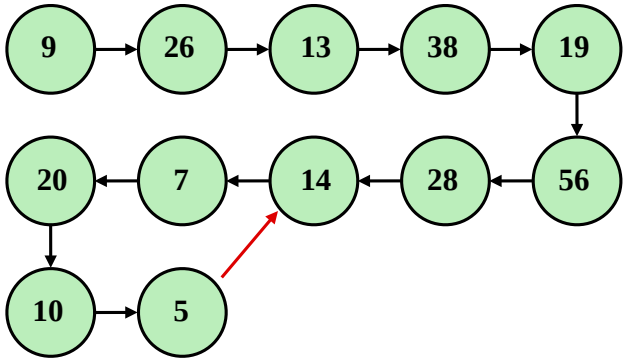
Answer 1

(i)



[4 marks]

(ii)



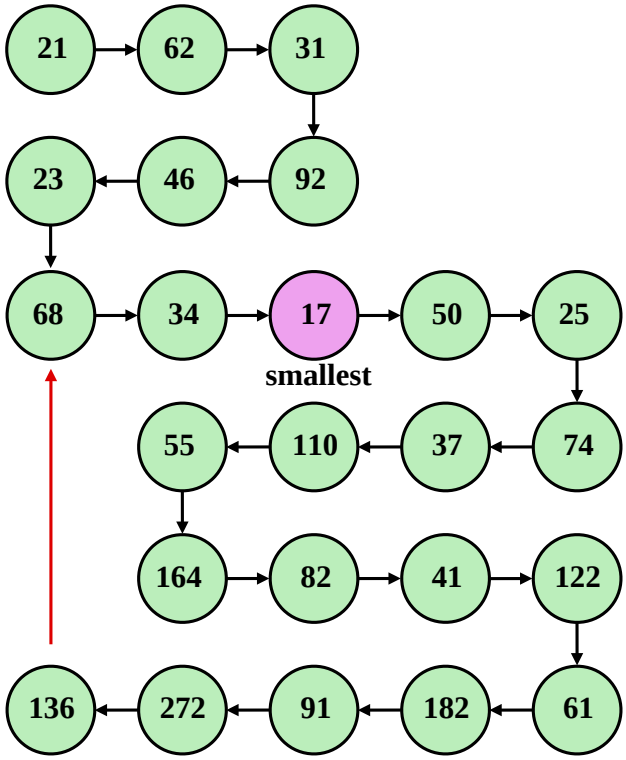
[4 marks]

(iii) Answer 1 exhibits behaviour like the regular Collatz sequence we've previously looked at falling into a loop with 1 as smallest number.
Call this the 1-loop.
However, answer 2 shows that a Collatz like conjecture for the sign flipped sequence would not be true as this sequence never gets to 1.
We've exhibited a number, 9, that does not “fall to earth”.
Instead it falls into a loop where 5 is the smallest number.
Call this the 5-loop.

[2 marks]

Answer 2

(i)



[6 marks]

(ii) The 21 falls into another, different, loop.
The smallest number in the loop is 17, so call it the 17-loop.
The equivalent of the Collatz conjecture is (again) NOT true for this iteration.
Numbers seem to either fall into the 1-loop, or 5-loop or 17-loop.
It's not currently known if there are any other loops for this iteration.

[2 marks]

Extra : If a sign flipped Collatz 100 table is made, and entries coloured according to which loop they end up in the following results.

Sign Flipped Collatz Rule 100 Table

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

There does not seem to be any obvious pattern...

Answer 3

(i) $5 \Rightarrow 22 \Rightarrow 11 \Rightarrow 52 \Rightarrow 26 \Rightarrow 13 \Rightarrow 62 \Rightarrow 31 \Rightarrow 152 \Rightarrow 76 \Rightarrow 38 \Rightarrow 19 \Rightarrow 92 \Rightarrow 46 \Rightarrow 23 \Rightarrow 112 \Rightarrow 56 \Rightarrow 28 \Rightarrow 14 \Rightarrow 7 \Rightarrow 32 \Rightarrow 16 \Rightarrow 8 \Rightarrow 4 \Rightarrow$ [2, 1 loop]

[4 marks]

(ii) $15 \Rightarrow 72 \Rightarrow 36 \Rightarrow 18 \Rightarrow 9 \Rightarrow 42 \Rightarrow 21 \Rightarrow 102 \Rightarrow 51 \Rightarrow 252 \Rightarrow 126 \Rightarrow 63 \Rightarrow 312 \Rightarrow 156 \Rightarrow 78 \Rightarrow 39 \Rightarrow 192 \Rightarrow 96 \Rightarrow 48 \Rightarrow 24 \Rightarrow$ [12, 6, 3 loop]

[4 marks]

(iii) An initial investigation suggests that the pattern may continue to hold. However, some numbers give sequences that include some numerically large integers. Sequences were terminated if a term greater than 2,147,483,647 occurred.
So, there is more work to do on this !

Watts' 100 Table

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

[12 marks]

Extra :

Pick an iterative rule that is similar to that of the Collatz conjecture and explore it, initially on paper.
Your rule should be of the form,

$$y = \begin{cases} \frac{x}{2} & \text{if } x \text{ is even} \\ a\,x \pm b & \text{if } x \text{ is odd} \end{cases}$$

for some integer values of a and b that you have chosen.
You also get to pick if the sign is + or –

ITERATION

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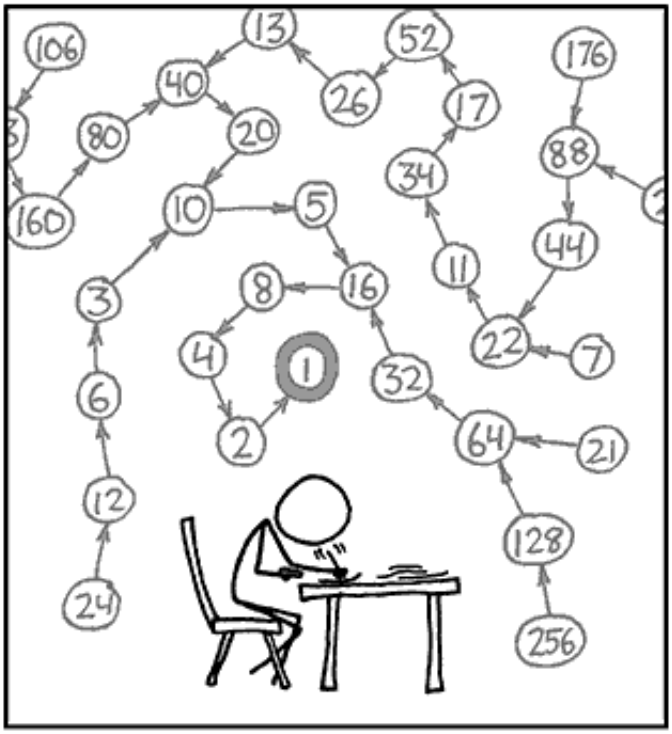

THE COLLATZ CONJECTURE STATES THAT IF YOU PICK A NUMBER, AND IF IT'S EVEN DIVIDE IT BY TWO AND IF IT'S ODD MULTIPLY IT BY THREE AND ADD ONE, AND YOU REPEAT THIS PROCEDURE LONG ENOUGH, EVENTUALLY YOUR FRIENDS WILL STOP CALLING TO SEE IF YOU WANT TO HANG OUT.

Detailed worked solutions to the exercises are available from m.hamsen2@unimail.derby.ac.uk