

3.6 Answers

GCSE Mathematics Iteration

3.6.1 Solution to 3.1 The Button Marked “Ans”

A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8
2	4	8	16	32	64	128	256

[4 marks]

3.6.2 Solution to 3.2 The Position-to-Term Rule

$$A_n = 2^n$$

$$A_{10} = 1024$$

[2 marks]

3.6.3 Solution to 3.3 The Term-to-Term Rule

$$A_1 = 2, \quad A_{n+1} = 2 A_n$$

- Program your calculator's answer button with the number 2 by tping in;
- 2 =
- Now type in;
- $2 \times \text{Ans} = = = = =$

(Count that you press the = key nine times to get terms 2, 3, 4, 5, 6, 7, 8, 9, 10

$$A_{10} = 1024$$

[2 marks]

3.6.4 Solution to 3.4 Example

(i)

$$B_1 = 1, \quad B_{n+1} = \frac{1}{2} B_n + 1$$

B_1	B_2	B_3	B_4	B_5	B_6	B_7	B_8
1	$\frac{3}{2}$	$\frac{7}{4}$	$\frac{15}{8}$	$\frac{31}{16}$	$\frac{63}{32}$	$\frac{127}{64}$	$\frac{255}{128}$

[4 marks]

(ii)

$$B_n = \frac{2^n - 1}{2^{n-1}}$$

[4 marks]

3.6.5 Solution to 3.5 Exercise

Answer 1

(i)

T_1	T_2	T_3	T_4	T_5	T_6	T_7	T_8
3	9	27	81	243	729	2187	6561

[4 marks]

(ii) The position-to-term rule is $T_n = 3^n$

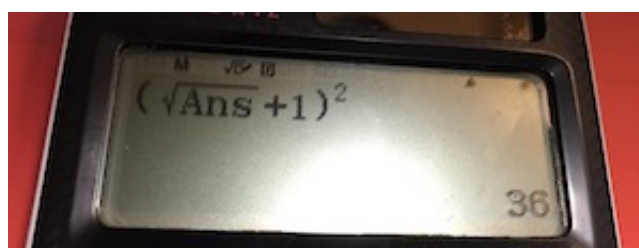
[2 marks]

(iii) The term-to-term rule is $T_1 = 3$, $T_{n+1} = 3 T_n$

[2 marks]

Answer 2

$$D_1 = 1 \quad D_{n+1} = (\sqrt{D_n} + 1)^2$$



(i)

D_1	D_2	D_3	D_4	D_5	D_6	D_7	D_8
1	4	9	16	25	36	49	64

[4 marks]

(ii) The position-to-term rule is $D_n = n^2$

[2 marks]

(iii) Square numbers

[1 mark]

Answer 3

(i) $P_1 = 1 \quad P_{n+1} = 3 - P_n$

P_1	P_2	P_3	P_4	P_5	P_6
1	2	1	2	1	2

[4 marks]

(ii) $P_{100} = 2$

[2 marks]

(iii) $Q_1 = 3, \quad Q_{n+1} = 3 - Q_n$

Q_1	Q_2	Q_3	Q_4	Q_5	Q_6
3	0	3	0	3	0

[4 marks]

(iv) $Q_{1000} = 0$

[2 marks]

(v) $R_1 = \frac{1}{2}, \quad R_{n+1} = 3 - R_n$

R_1	R_2	R_3	R_4	R_5	R_6
$\frac{1}{2}$	$\frac{5}{2}$	$\frac{1}{2}$	$\frac{5}{2}$	$\frac{1}{2}$	$\frac{5}{2}$

[4 marks]

(vi) $R_{1000000} = \frac{5}{2}$

[2 marks]

(vii) Let $R_1 = x$ in which case $R_2 = 3 - x$

Then $R_3 = 3 - (3 - x)$

$= 3 - 3 + x$

$= x$ which proves that the sequence is alternating

[4 marks]

Answer 4

$$U_1 = 1 \quad U_{n+1} = \frac{2}{3} U_n + 1$$

(i)

U_1	U_2	U_3	U_4	U_5	U_6
1	$\frac{5}{3}$	$\frac{19}{9}$	$\frac{65}{27}$	$\frac{211}{81}$	$\frac{665}{243}$

[4 marks]

(ii) $U_n = \frac{3^n - 2^n}{3^n - 1} \Rightarrow U_6 = \frac{3^6 - 2^6}{3^6 - 1} = \frac{729 - 64}{3^5} = \frac{665}{243}$

[2 marks]

Answer 5

$$x_0 = 2$$

$$x_1 = 1.817120593$$

$$x_2 = 1.853318496$$

$$x_3 = 1.846265953$$

[3 marks]

Answer 6

(a) $P_1 = 200$ end of March

$P_2 = 190$ end of April

$P_3 = 178$ end of May

$P_4 = 164$ end of June

[3 marks]

(b) The rabbit population on the farm will decline to zero.

[1 mark]