4.4 Answers

4.4.1 Solution to 4.1 "Doing The Same to Both Sides"

$$x = \frac{x + 20}{5}$$

$5x = x + 20$ Multiply both sides by 5

$4x = 20$ Subtract x from both sides

$x = 5$ Divide both sides by 4

[3 marks]

4.4.2 Solution to 4.2 An Iterative Solution

(i) $A_1 = 10, \quad A_{n+1} = \dfrac{A_n + 20}{5}$

Term	Value
A_1	10
A_2	6
A_3	5.2
A_4	5.04
A_5	5.008
A_6	5.0016
A_7	5.00032
A_8	5.000064

[4 marks]

(ii) The limit of this iterative sequence seems to be 5

[1 mark]

(iii) Let $A_1 = 5$ in which case $A_2 = \dfrac{5 + 20}{5} = \dfrac{25}{5} = 5$

So 5 is indeed a fixed point of the iteration.

[1 mark]

(iv) Conclusion

A solution to the equation $x = \dfrac{x + 20}{5}$ is $x = 5$

[1 mark]

4.4.3 Solutions to 4.3 Exercise

Answer 1

(i) $B_1 = 10, \quad B_{n+1} = \dfrac{B_n + 6}{2}$

Term	Value
B_1	10
B_2	8
B_3	7
B_4	6.5
B_5	6.25
B_6	6.125
B_7	6.0625
B_8	6.03125

[4 marks]

(ii) The limit of this iterative sequence seems to be 6

[1 mark]

(iii) Let $B_1 = 6$ in which case $B_2 = \dfrac{6 + 6}{2} = \dfrac{12}{2} = 6$

So 6 is indeed a fixed point of the iteration.

[1 mark]

(iv) Conclusion

A solution to the equation $x = \dfrac{x + 6}{2}$ is $x = 6$

[1 mark]

Answer 2

Note : I made a small edit to this examination question to stop pupils getting the correct answer for the wrong reason.

$P_1 = 400$ start of year 1

$P_2 = 440$ start of year 2

$P_3 = 484$ start of year 3

[2 marks]

Answer 3

(i) $C_1 = 10, \quad C_{n+1} = 6 - \dfrac{8}{C_n}$

Term	Value
C_1	10
C_2	5.2
C_3	4.461538462
C_4	4.206896552
C_5	4.098360656
C_6	4.048000000
C_7	4.023715415
C_8	4.011787819

[4 marks]

(ii) The limit of this iterative sequence seems to be 4

[1 mark]

(iii) Let $C_1 = 4$ in which case $C_2 = 6 - \dfrac{8}{4} = 4$

So, 4 is indeed a fixed point of the iteration.

[1 mark]

(iv) Conclusion

A solution to the equation $x = 6 - \dfrac{8}{x}$ is $x = 4$

[1 mark]

(v) The other positive integer fixed point is $x = 2$

because if $C_1 = 2$ then $C_2 = 6 - \dfrac{8}{2} = 2$

[3 marks]

Answer 4

(i) $D_1 = 10, \quad D_{n+1} = \dfrac{18}{D_n} - 7$

Term	Value
D_1	10
D_2	– 5.2
D_3	– 10.46153846
D_4	– 8.720588235
D_5	– 9.064080944
D_6	– 8.985860465
D_7	– 9.003147063
D_8	– 8.999300897

[4 marks]

(ii) The limit of this iterative sequence seems to be – 9

[1 mark]

(iii) Let $D_1 = -9$ in which case $D_2 = \dfrac{18}{(-9)} - 7 = -9$

So, -9 is indeed a fixed point of the iteration.

[1 mark]

(iv) Conclusion

A solution to the equation $x = \dfrac{18}{x} - 7$ is $x = -9$

[1 mark]

(v) The positive integer fixed point is $x = 2$

because if $D_1 = 2$ then $D_2 = \dfrac{18}{2} - 7 = 2$

[3 marks]

Answer 5

(i) $E_1 = 10, \quad E_{n+1} = -\left(\dfrac{15}{E_n} + 8\right)$

Term	Value
E_1	10
E_2	– 9.5
E_3	– 6.421052632
E_4	– 5.663934426
E_5	– 5.351664255
E_6	– 5.197133586
E_7	– 5.113793642
E_8	– 5.066756883

[4 marks]

(ii) The limit of this iterative sequence seems to be – 5

[1 mark]

(iii) Let $E_1 = -5$ in which case $E_2 = -\left(\dfrac{15}{(-5)} + 8\right)$

$$= -(-3 + 8)$$

$$= -5$$

So, -5 is indeed a fixed point of the iteration.

[1 mark]

(iv) Conclusion

A solution to the equation $x = -\left(\dfrac{15}{x} + 8\right)$ is $x = -5$

[1 mark]

(v) The other negative integer fixed point is $x = -3$

because if $E_1 = -3$ then $E_2 = -\left(\dfrac{15}{(-3)} + 8\right)$

$$= -(-5 + 8)$$

$$= -3$$

[3 marks]

Answer 6

$$F_{n+1} = \frac{9}{(F_n)^2} - \frac{3}{F_n} - 5$$

(i) Let $F_1 = 1$ in which case $F_2 = \frac{9}{1^2} - \frac{3}{1} - 5$

$= 9 - 3 - 5$

$= 1$

So, 1 is indeed a fixed point of the iteration.

[2 marks]

(ii) Let $F_1 = -3$ in which case $F_2 = \frac{9}{(-3)^2} - \frac{3}{(-3)} - 5$

$= 1 + 1 - 5$

$= -3$

So, -3 is indeed a fixed point of the iteration.

[2 marks]

(iii)

Term	Value
F_1	10
F_2	– 5.21
...	...
F_{10}	– 3.245882962
...	...
F_{50}	– 3.04568015
...	...
F_{100}	– 3.022658768

[7 marks]