5.5 Answers

5.5.1 Solution to 5.2 By Algebra

$$x = \frac{(1 - x)}{2}$$

$2x = 1 - x$ Multiply both sides by 2

$3x = 1$ Add x to both sides

$x = \frac{1}{3}$ Divide both sides by 3

[3 marks]

5.5.2 Solution to 5.3 By Iteration

(i) $A_1 = 1, \; A_{n+1} = \frac{(1 - A_n)}{2}$

Term	Value
A_1	1
A_2	0
A_3	0.5
A_4	0.25
...	...
A_{10}	0.33203125
...	...
A_{20}	0.3333320618

[4 marks]

(ii) The limit of this iterative sequence seems to be $\frac{1}{3}$

[1 mark]

(iii) Let $A_1 = \frac{1}{3}$ in which case

$$A_2 = \frac{\left(1 - \frac{1}{3}\right)}{2} \times \frac{3}{3}$$

$$= \frac{3 - 1}{2 \times 3}$$

$$= \frac{2}{2 \times 3}$$

$$= \frac{1}{3}$$

So, $\frac{1}{3}$ is indeed a fixed point of the iteration

[3 marks]

5.5.3 Solutions to 5.4 Exercise

Answer 1

(i) $B_1 = 1,\ B_{n+1} = \dfrac{(1 + B_n)}{4}$ complete the table,

Term	Value
B_1	1
B_2	0.5
B_3	0.375
B_4	0.34375
...	...
B_{10}	0.3333358765
...	...
B_{20}	0.3333333333

[4 marks]

(ii) The limit of this iterative sequence seems to be $\dfrac{1}{3}$

[1 mark]

(iii) Let $B_1 = \dfrac{1}{3}$ in which case

$$B_2 = \frac{\left(1 + \frac{1}{3}\right)}{4} \times \frac{3}{3}$$

$$= \frac{3 + 1}{4 \times 3}$$

$$= \frac{4}{4 \times 3}$$

$$= \frac{1}{3}$$

So, $\dfrac{1}{3}$ is indeed a fixed point of the iteration

[3 marks]

Answer 2

Let $C_1 = \frac{3}{7}$ in which case

$$C_2 = \frac{\left(3 - \frac{6}{7}\right)}{5} \times \frac{7}{7}$$

$$= \frac{21 - 6}{35}$$

$$= \frac{15}{35}$$

$$= \frac{3}{7}$$

So, $\frac{3}{7}$ is indeed a fixed point of the iteration

[3 marks]

Answer 3

(i) Complete the table to show what sevenths look like, expressed as decimals.

Vulgar Fraction	Decimal Fraction
$\frac{1}{7}$	0.1428571429...
$\frac{2}{7}$	0.2857142857...
$\frac{3}{7}$	0.4285714286...
$\frac{4}{7}$	0.5714285714...
$\frac{5}{7}$	0.7142857143...
$\frac{6}{7}$	0.8571428571...

[2 marks]

(ii) $D_1 = 1,\ D_{n+1} = \dfrac{(3 + D_n)}{8}$

Term	Value
D_1	1
D_2	0.5
D_3	0.4375
D_4	0.4296875
...	...
D_{10}	0.4285714328

[4 marks]

(iii) The limit of this iterative sequence seems to be $\frac{3}{7}$

[1 mark]

(iv) Let $D_1 = \frac{3}{7}$ in which case

$$D_2 = \frac{\left(3 + \frac{3}{7}\right)}{8} \times \frac{7}{7}$$

$$= \frac{21 + 3}{8 \times 7}$$

$$= \frac{24}{8 \times 7}$$

$$= \frac{3}{7}$$

So, $\frac{3}{7}$ is indeed a fixed point of the iteration

[3 marks]

Answer 4

(i) $E_1 = 1, \quad E_{n+1} = \dfrac{(5 - 4E_n)}{3}$

Term	Value
E_1	1
E_2	0.333333333...
E_3	1.222222222...
E_4	0.03703703704...
...	...
E_{10}	– 3.090941422...
...	...
E_{20}	– 66.85789754...
...	...
E_{40}	– 21307.28548...

[4 marks]

(ii) "not convergent" means that the iteration is not settling down to give a fixed point.

[1 mark]

(iii) Let $E_1 = \dfrac{5}{7}$ in which case

$$D_2 = \frac{\left(5 - \frac{4\times 5}{7}\right)}{3} \times \frac{7}{7}$$

$$= \frac{35 - 20}{3 \times 7}$$

$$= \frac{15}{3 \times 7}$$

$$= \frac{5}{7}$$

So, $\dfrac{5}{7}$ is indeed a fixed point of the iteration

[3 marks]

Answer 5

(i) For getting the iteration into calculator

[1 mark]

(ii) With $F_1 = 10$ and $F_{n+1} = \dfrac{\left(F_n + \frac{3}{5}\right)}{4}$ complete the table,

Term	Value
F_1	10
F_2	2.65
F_3	0.8125
F_4	0.353125
...	...
F_{10}	0.200037384
...	...
F_{20}	0.2

[4 marks]

(iii) The limit of this iterative sequence seems to be $\dfrac{1}{5}$

[1 mark]

(iv) Let $F_1 = \dfrac{1}{5} \Rightarrow F_2 = \dfrac{\left(\frac{1}{5} + \frac{3}{5}\right)}{4} \times \dfrac{5}{5} = \dfrac{1+3}{20} = \dfrac{4}{20} = \dfrac{1}{5}$

So, $\dfrac{1}{5}$ is indeed a fixed point of the iteration.

[4 marks]

(v) Conclusion

A solution to the equation $x = \dfrac{\left(x + \frac{3}{5}\right)}{4}$ is $x = \dfrac{1}{5}$

[1 mark]