VULGAR FRACTIONS

Year 6 (England)
Primary 6 (Scotland)

What is a vulgar fraction ?

A vulgar fraction is one that has a numerator and a denominator.

Formally, a vulgar fraction is of the form $\frac{P}{Q}$ where P and Q are integers, $Q \neq 0$.

They are not to be confused with decimal fractions which feature a decimal point.

Lesson 1

Year 6, Primary 6, Mathematics
Vulgar Fractions

W.O.M.B.A.T

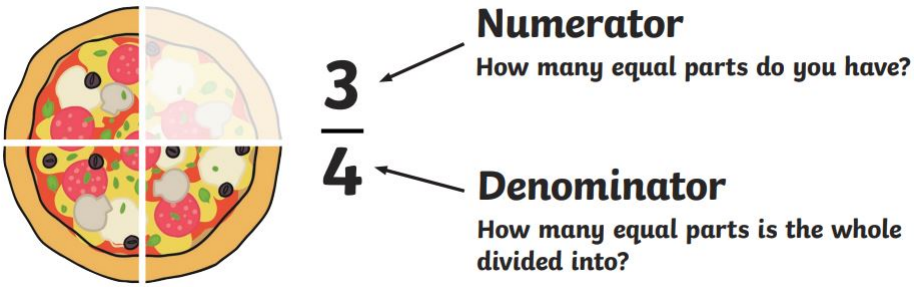
Wizards' Ordinary Mathematics and Basic Aptitude Test

1.1 Easy Adds

Some fractions are composed of two parts,

- an upper number called the numerator
- a lower number called the denominator

For example,



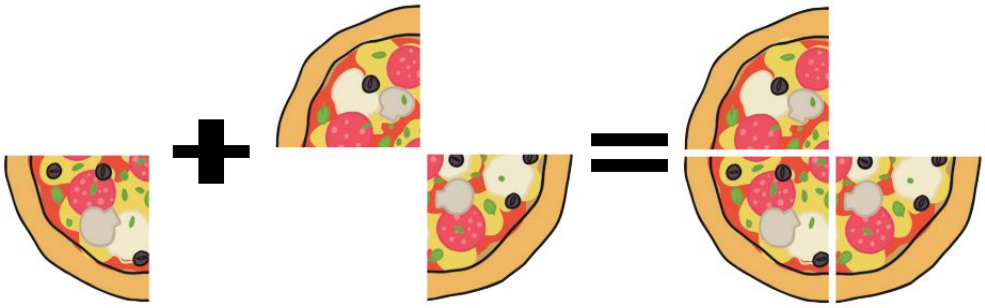
If two fractions have the same denominator then adding them is easy.

Calculate, $\frac{1}{4} + \frac{2}{4} =$

In words this question asked,

"What is one quarter of a pizza added to two quarters of a pizza ?"

And the answer given can be checked by drawing pizza,



To remember which is Numerator and which is **D**enominator: "**D** is **D**own".

1.2 Exercise

Marks Available : 12

Question 1

Write down a fraction which has a numerator of 5 and a denominator of 7.

[1 mark]

Question 2

Consider the fraction $\frac{7}{8}$

(i) Is the 7 called the numerator or the generator ?

[1 mark]

(ii) Is the 8 called the imitator or the denominator ?

[1 mark]

Question 3

What is $\frac{1}{5}$ of a pizza added to $\frac{2}{5}$ of a pizza ?

[2 marks]

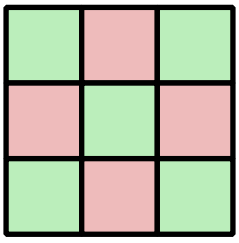
Question 4

What is $\frac{2}{8}$ of a cake added to $\frac{3}{8}$ of a cake ?

[2 marks]

Question 5

A slice of Battenberg cake is made of 9 equally sized square pieces.



Billy eats 4 pieces and Eddy eats 3.

(i) What fraction of the cake has been eaten ?

[2 marks]

(ii) Did Eddy eat $\frac{1}{2}$ the cake or $\frac{1}{3}$ of the cake ?

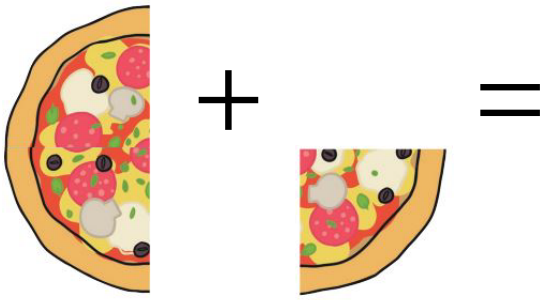
Explain your answer.

[3 marks]

1.3 Different Denominators

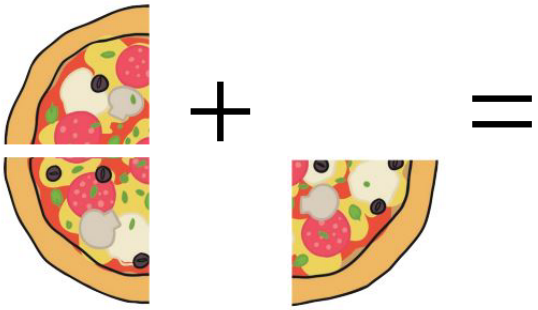
Adding two fractions which have different denominators is a new problem.
We solve it by turning it into the sort of problem we already know how to solve.
For example, suppose you are asked,

What is half a pizza added to a quarter of a pizza ?



We want to work out what $\frac{1}{2} + \frac{1}{4}$ is equal to.

The key idea is to write the half as two quarters, because then,



So, $\frac{\mathbf{1}}{\mathbf{2}} + \frac{1}{4} = \frac{\mathbf{2}}{\mathbf{4}} + \frac{1}{4}$ which we know how to work out.

Mathematically, to do this without having to draw lots of pizza all the time, we make use of two magical property of the number 1.

The first is that,

$\Rightarrow \Rightarrow \Rightarrow$ **Any number multiplied by 1 is itself** $\Leftarrow \Leftarrow \Leftarrow$

For example.

$$7 \times 1 = 7, \qquad 23 \times 1 = 23, \qquad \frac{2}{3} \times 1 = \frac{2}{3}$$

The second piece of wizardry is that,

$\Rightarrow \Rightarrow \Rightarrow$ **Any number divided by itself is one** $\Leftarrow \Leftarrow \Leftarrow$

For example,

$$\frac{2}{2} = 1, \qquad \frac{3}{3} = 1, \qquad \frac{4}{4} = 1, \qquad \frac{5}{5} = 1$$

We actually are going to use this backwards and replace a 1 with whichever of these helps us most. This is a little tricky to get the hang of so you may want your teacher to walk you through the two examples on the next page.

Example #1

Showing all the steps, work out $\frac{1}{2} + \frac{1}{8}$

$\frac{1}{2} + \frac{1}{4} = \frac{1}{2} \times \mathbf{1} + \frac{1}{4}$ Step 1: Multiply by 1

$= \frac{1}{2} \times \frac{\mathbf{2}}{\mathbf{2}} + \frac{1}{4}$ Step 2: Write 1 in a more useful way

$= \frac{2}{4} + \frac{1}{4}$ Step 3: Do the multiplication

$= \frac{3}{4}$ Step 4: Do the common denominator addition

Example #2

Showing all the steps, work out $\frac{2}{5} + \frac{7}{15}$

$\frac{2}{5} + \frac{7}{15} = \frac{2}{5} \times \mathbf{1} + \frac{7}{15}$ Step 1: Multiply by 1

$= \frac{2}{5} \times \frac{\mathbf{3}}{\mathbf{3}} + \frac{7}{15}$ Step 2: Write 1 in a more useful way

$= \frac{6}{15} + \frac{7}{15}$ Step 3: Do the multiplication

$= \frac{13}{15}$ Step 4: Do the common denominator addition

1.4 Exercise

Marks Available : 25

Question 1

Showing all the steps, work out $\frac{1}{2} + \frac{1}{8}$

[4 marks]

Question 2

Showing all the steps, work out $\frac{2}{5} + \frac{3}{10}$

[4 marks]

Question 3

Showing all the steps, work out $\frac{2}{3} + \frac{1}{9}$

[4 marks]

Question 4

Showing all the steps, work out $\frac{3}{5} + \frac{7}{20}$

[4 marks]

Question 5

Work out, $\frac{1}{2} - \frac{1}{8}$

[4 marks]

Question 6

Work our, $\frac{4}{5} - \frac{8}{15}$

[5 marks]

1.5 Answers

1.5.1 Solution to 1.2 Exercise

Answer 1

$\frac{5}{7}$

Answer 2

(i) 7 is the numerator

(ii) 8 is the denominator

Answer 3

$\frac{3}{5}$ of a pizza

[1, 1, 1, 2 marks]

Answer 4

$\frac{5}{8}$ of a cake

Answer 5

(i) $\frac{7}{9}$ of the cake

(ii) $\frac{1}{3}$ because $\frac{3}{9} = \frac{1}{3}$

[2, 2, 3 marks]

1.5.2 Solution to 1.4 Exercise

Answer 1

$$\frac{1}{2} + \frac{1}{8} = \frac{1}{2} \times \mathbf{1} + \frac{1}{8}$$ Step 1: Multiply by 1

$$= \frac{1}{2} \times \frac{\mathbf{4}}{\mathbf{4}} + \frac{1}{8}$$ Step 2: Write 1 in a more useful way

$$= \frac{4}{8} + \frac{1}{8}$$ Step 3: Do the multiplication

$$= \frac{5}{8}$$ Step 4: Do the common denominator addition

[4 marks]

Answer 2

$$\frac{2}{5} + \frac{3}{10} = \frac{2}{5} \times \mathbf{1} + \frac{3}{10}$$ Step 1: Multiply by 1

$$= \frac{2}{5} \times \frac{\mathbf{2}}{\mathbf{2}} + \frac{3}{10}$$ Step 2: Write 1 in a more useful way

$$= \frac{4}{10} + \frac{3}{10}$$ Step 3: Do the multiplication

$$= \frac{7}{10}$$ Step 4: Do the common denominator addition

[4 marks]

Answer 3

$$\frac{2}{3} + \frac{1}{9} = \frac{2}{3} \times \mathbf{1} + \frac{1}{9}$$ Step 1: Multiply by 1

$$= \frac{2}{3} \times \frac{\mathbf{3}}{\mathbf{3}} + \frac{1}{9}$$ Step 2: Write 1 in a more useful way

$$= \frac{6}{9} + \frac{1}{9}$$ Step 3: Do the multiplication

$$= \frac{7}{9}$$ Step 4: Do the common denominator addition

[4 marks]

Answer 4

$$\frac{3}{5} + \frac{7}{20} = \frac{3}{5} \times \mathbf{1} + \frac{7}{20}$$ Step 1: Multiply by 1

$$= \frac{3}{5} \times \frac{\mathbf{4}}{\mathbf{4}} + \frac{7}{20}$$ Step 2: Write 1 in a more useful way

$$= \frac{12}{20} + \frac{7}{20}$$ Step 3: Do the multiplication

$$= \frac{19}{20}$$ Step 4: Do the common denominator addition

[4 marks]

Answer 5

$$\frac{1}{2} - \frac{1}{8} = \frac{1}{2} \times \mathbf{1} - \frac{1}{8}$$ Step 1: Multiply by 1

$$= \frac{1}{2} \times \frac{\mathbf{4}}{\mathbf{4}} - \frac{1}{8}$$ Step 2: Write 1 in a more useful way

$$= \frac{4}{8} - \frac{1}{8}$$ Step 3: Do the multiplication

$$= \frac{3}{8}$$ Step 4: Do the common denominator subtraction

[4 marks]

Answer 6

$$\frac{4}{5} - \frac{8}{15} = \frac{4}{5} \times \mathbf{1} - \frac{8}{15}$$ Step 1: Multiply by 1

$$= \frac{4}{5} \times \frac{\mathbf{3}}{\mathbf{3}} - \frac{8}{15}$$ Step 2: Write 1 in a more useful way

$$= \frac{12}{15} - \frac{8}{15}$$ Step 3: Do the multiplication

$$= \frac{4}{15}$$ Step 4: Do the common denominator subtraction

[5 marks]

Lesson 2

Year 6, Primary 6, Mathematics
Vulgar Fractions

Be Wizard At Maths

I don't usually make jokes about fractions...
...but I will make one if I halve two.

2.1 Lowest Common Multiple

The multiples of 6 are the numbers that are in the 6 times table.

$$1 \times 6 = 6$$

$$2 \times 6 = 12$$

$$3 \times 6 = 18$$

$$4 \times 6 = 24$$

$$5 \times 6 = 30$$

$$6 \times 6 = 42$$

$$7 \times 6 = 48$$

$$\ldots \times \ldots = \ldots$$

{Multiples of 6} = { 6, 12, 18, 24, 30, 36, 42, 48, 54, 60, 66, 72, ... }

The multiples of 15 are the numbers that are in the 15 times table.

$$1 \times 15 = 15$$

$$2 \times 15 = 30$$

$$3 \times 15 = 45$$

$$\ldots \times \ldots = \ldots$$

{Multiples of 15} = {15, 30, 45, ...}

The ***lowest common multiple*** of 6 and 15 is the first integer that is in the 6 times table and also in the 15 times table

$$lcm\ \{6,15\} = 30$$

This is the key to adding awkward fractions which have awkward denominators.

For example, $\frac{1}{6} + \frac{4}{15}$

The trick is to work out the ***lowest common multiple*** of the denominators and then use the multiplying by 1 technique on ***both*** pieces of the fraction sum.

$$\frac{1}{6} + \frac{4}{15} = \frac{1}{6} \times \mathbf{1} + \frac{4}{15} \times \mathbf{1} \qquad \text{Step 1: Multiply by 1}$$

$$= \frac{1}{6} \times \frac{\mathbf{5}}{\mathbf{5}} + \frac{4}{15} \times \frac{\mathbf{2}}{\mathbf{2}} \qquad \text{Step 2: Use } lcm\{6,15\} \text{ is 30}$$

$$= \frac{5}{30} + \frac{8}{30} \qquad \text{Step 3: Do the two multiplications}$$

$$= \frac{13}{30} \qquad \text{Step 4: Do the easy addition}$$

The ***lowest common multiple*** when used in this way is often called the ***lowest common denominator***

2.2 Exercise

Marks Available : 40

Question 1

(i) List the first six multiples of 8

[1 mark]

(ii) List the first six multiples of 12

[1 mark]

(iii) What is $lcm\{8,12\}$?

[1 mark]

(iv) Showing all the steps, work out $\frac{3}{8} + \frac{5}{12}$

[7 marks]

Question 2

(i) List the first six multiples of 10

[1 mark]

(ii) List the first six multiples of 25

[1 mark]

(iii) What is *lcm*{10,25} ?

[1 mark]

(iv) Showing all the steps, work out $\frac{3}{10} + \frac{7}{25}$

[7 marks]

Question 3

(i) List the first ten multiples of 9

[1 mark]

(ii) List the first six multiples of 24

[1 mark]

(iii) What is *lcm*{9,24} ?

[1 mark]

(iv) Showing all the steps, work out $\frac{2}{9} + \frac{7}{24}$

[7 marks]

Question 4

(i) List the first eight multiples of 12

[1 mark]

(ii) List the first eight multiples of 14

[1 mark]

(iii) What is $lcm\{12,14\}$?

[1 mark]

(iv) Showing all the steps, work out $\frac{5}{12} - \frac{1}{14}$

[7 marks]

2.3 Answers

2.3.1 Solution to 2.2 Exercise

Answer 1

(i) {Multiples of 8} = {8, 16, **24**, 32, 40, 48, ...}

[1 mark]

(ii) {Multiples of 12} = {12, **24**, 36, 48, 60, 72, ...}

[1 mark]

(iii) *lcm*{8, 12} = 24

[1 mark]

(iv)

$$\frac{3}{8} + \frac{5}{12} = \frac{3}{8} \times \mathbf{1} + \frac{5}{12} \times 1 \qquad \text{Step 1: Multiply by 1}$$

$$= \frac{3}{8} \times \frac{\mathbf{3}}{\mathbf{3}} + \frac{5}{12} \times \frac{\mathbf{2}}{\mathbf{2}} \qquad \text{Step 2: Use } lcm\{8,12\} \text{ is 24}$$

$$= \frac{9}{24} + \frac{10}{24} \qquad \text{Step 3: Do the two multiplications}$$

$$= \frac{19}{24} \qquad \text{Step 4: Do the easy addition}$$

[7 marks]

Answer 2

(i) {Multiples of 10} = {10, 20, 30, 40, **50**, 60, ...}

[1 mark]

(ii) {Multiples of 25} = {25, **50**, 75, 100, 125, 150, ...}

[1 mark]

(iii) *lcm*{10, 25} = 50

[1 mark]

(iv)

$$\frac{3}{10} + \frac{7}{25} = \frac{3}{10} \times \mathbf{1} + \frac{7}{25} \times 1 \qquad \text{Step 1: Multiply by 1}$$

$$= \frac{3}{10} \times \frac{\mathbf{5}}{\mathbf{5}} + \frac{7}{25} \times \frac{\mathbf{2}}{\mathbf{2}} \qquad \text{Step 2: Use } lcm\{10,25\} \text{ is 50}$$

$$= \frac{15}{50} + \frac{14}{50} \qquad \text{Step 3: Do the two multiplications}$$

$$= \frac{29}{50} \qquad \text{Step 4: Do the easy addition}$$

[7 marks]

Answer 3

(i) {Multiples of 9} = {9, 18, 27, 36, 45, 54, 63, **72**, 81, 90, ...}

[1 mark]

(ii) {Multiples of 24} = {24, 48, **72**, 96, ...}

[1 mark]

(iii) $lcm\{9, 24\} = 72$

[1 mark]

(iv)

$$\frac{2}{9} + \frac{7}{24} = \frac{2}{9} \times \mathbf{1} + \frac{7}{24} \times 1 \qquad \text{Step 1: Multiply by 1}$$

$$= \frac{2}{9} \times \frac{\mathbf{8}}{\mathbf{8}} + \frac{7}{24} \times \frac{\mathbf{3}}{\mathbf{3}} \qquad \text{Step 2: Use } lcm\{9,24\} \text{ is 72}$$

$$= \frac{16}{72} + \frac{21}{72} \qquad \text{Step 3: Do the two multiplications}$$

$$= \frac{37}{72} \qquad \text{Step 4: Do the easy addition}$$

[7 marks]

Answer 4

(i) {Multiples of 12} = {12, 24, 36, 48, 60, 72, **84**, 96, ...}

[1 mark]

(ii) {Multiples of 14} = {14, 28, 42, 56, 70, **84**, 98, 112, ...}

[1 mark]

(iii) $lcm\{12, 14\} = 84$

[1 mark]

(iv)

$$\frac{5}{12} - \frac{1}{14} = \frac{5}{12} \times \mathbf{1} - \frac{1}{14} \times 1 \qquad \text{Step 1: Multiply by 1}$$

$$= \frac{5}{12} \times \frac{\mathbf{7}}{\mathbf{7}} - \frac{1}{14} \times \frac{\mathbf{6}}{\mathbf{6}} \qquad \text{Step 2: Use } lcm\{12,14\} \text{ is 84}$$

$$= \frac{35}{84} - \frac{6}{84} \qquad \text{Step 3: Do the two multiplications}$$

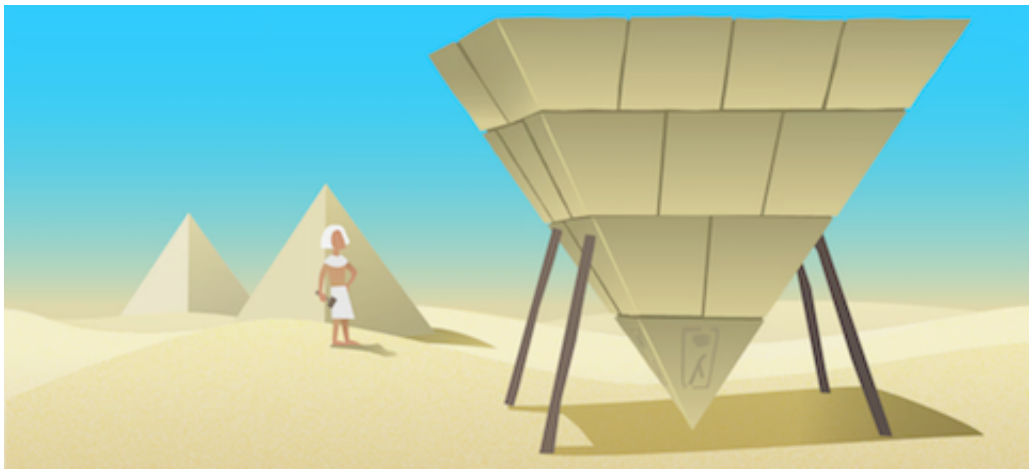
$$= \frac{29}{84} \qquad \text{Step 4: Do the easy addition}$$

[7 marks]

Lesson 3

Year 6, Primary 6, Mathematics
Vulgar Fractions

Top Heavy Fractions



Having trouble understanding top heavy fractions ?
Our helpline is open 24/7

3.1 What is a top heavy fraction ?

A ***top heavy fraction*** (also called an ***improper fraction***) is one where the numerator is greater than the denominator.

For example, $\frac{24}{7}$ is top heavy.

They can be written as ***mixed number fractions*** by doing bus stop division.

Example #1

To write $\frac{98}{3}$ as a mixed number fraction do the following;

$$\frac{98}{3} \Rightarrow 3\overline{)98} \Rightarrow \begin{array}{r} 32\ r\ 2 \\ 3\overline{)98} \end{array} \Rightarrow 32 + \frac{2}{3}$$

$$\frac{98}{3} = 32\frac{2}{3}$$

Example #2

To write $\frac{69}{5}$ as a mixed number fraction do the following;

$$\frac{69}{5} \Rightarrow 5\overline{)69} \Rightarrow \begin{array}{r} 13\ r\ 4 \\ 5\overline{)6^{1}9} \end{array} \Rightarrow 13 + \frac{4}{5}$$

$$\frac{69}{5} = 13\frac{4}{5}$$

Being able to do the reverse process (turn a mixed number fraction to one that is top heavy) is also useful. The next two examples show how to do this.

Example #3

To write $32\frac{2}{3}$ as a top heavy fraction do the following;

$$32\frac{2}{3} = 32 + \frac{2}{3}$$

$$= \frac{32}{1} + \frac{2}{3} \qquad \text{Because } 32 = \frac{32}{1}$$

$$= \frac{32}{1} \times \mathbf{1} + \frac{2}{3} \qquad \text{Lesson 1 technique}$$

$$= \frac{32}{1} \times \frac{\mathbf{3}}{\mathbf{3}} + \frac{2}{3} \qquad \text{Lesson 1 technique}$$

$$= \frac{96}{3} + \frac{2}{3} \qquad \text{Lesson 1 technique}$$

$$= \frac{98}{3} \qquad \text{Lesson 1 technique}$$

Example #4

To write $13\frac{4}{3}$ as a top heavy fraction do the following;

$$13\frac{4}{5} = 13 + \frac{4}{5}$$

$$= \frac{13}{1} + \frac{4}{5} \qquad \text{Because } 13 = \frac{13}{1}$$

$$= \frac{13}{1} \times \mathbf{1} + \frac{4}{5} \qquad \text{Lesson 1 technique}$$

$$= \frac{13}{1} \times \frac{\mathbf{5}}{\mathbf{5}} + \frac{4}{5} \qquad \text{Lesson 1 technique}$$

$$= \frac{65}{5} + \frac{4}{5} \qquad \text{Lesson 1 technique}$$

$$= \frac{69}{5} \qquad \text{Lesson 1 technique}$$

3.2 Exercise

Marks Available : 16

Question 1

Write the following top heavy fractions as mixed number fractions.

(i) $\frac{7}{3}$ **(ii)** $\frac{56}{5}$

(iii) $\frac{93}{4}$ **(iv)** $\frac{31}{7}$

[8 marks]

Question 2

Write the following mixed number fractions as top heavy fractions.

(i) $4\frac{2}{5}$ **(ii)** $9\frac{3}{4}$

(iii) $50\frac{5}{6}$ **(iv)** $108\frac{1}{3}$

[8 marks]

3.3 Fast Conversion Formula

The mixed number fraction $a\frac{b}{c}$ has top heavy equivalent $\frac{ac + b}{c}$

Example #5

$$20\frac{3}{4} = \frac{20 \times 4 + 3}{4}$$
$$= \frac{80 + 3}{4}$$
$$= \frac{83}{4}$$

Example #6

$$5\frac{6}{7} = \frac{5 \times 7 + 6}{7}$$
$$= \frac{35 + 6}{7}$$
$$= \frac{41}{7}$$

3.4 Exercise

Marks Available : 30

Question 1

Use the Fast Conversion Formula to write as top heavy fractions;

(i) $8\frac{3}{11}$ **(ii)** $15\frac{2}{3}$

(iii) $7\frac{9}{10}$ **(iv)** $25\frac{3}{4}$

[8 marks]

Question 2

(i) List the first six multiples of 6

[1 mark]

(ii) List the first six multiples of 9

[1 mark]

(iii) What is *lcm*{6,9} ?

[1 mark]

(iv) Showing all the steps, work out $3\frac{5}{6} - 2\frac{5}{9}$

[8 marks]

Question 3

(i) List the first six multiples of 4

[1 mark]

(ii) List the first six multiples of 10

[1 mark]

(iii) What is $lcm\{4,10\}$?

[1 mark]

(iv) Showing all the steps, work out $5\frac{3}{4} + 3\frac{3}{10}$

[8 marks]

3.5 Answers

3.5.1 Solution to 3.2 Exercise

Answer 1

(i) $\frac{7}{3} \Rightarrow 3\overline{)7} \Rightarrow \begin{array}{r} 2 \\ 3\overline{)7} \end{array} r\ 1 \Rightarrow 2 + \frac{1}{3}$

$$\frac{7}{3} = 2\frac{1}{3}$$

[2 marks]

(ii) $\frac{56}{5} \Rightarrow 5\overline{)56} \Rightarrow \begin{array}{r} 11 \\ 5\overline{)56} \end{array} r\ 1 \Rightarrow 11 + \frac{1}{5}$

$$\frac{56}{5} = 11\frac{1}{5}$$

[2 marks]

(iii) $\frac{93}{4} \Rightarrow 4\overline{)93} \Rightarrow \begin{array}{r} 23 \\ 4\overline{)93} \end{array} r\ 1 \Rightarrow 23 + \frac{1}{4}$

$$\frac{93}{4} = 23\frac{1}{4}$$

[2 marks]

(iv) $\frac{31}{7} \Rightarrow 7\overline{)31} \Rightarrow \begin{array}{r} 04 \\ 7\overline{)31} \end{array} r\ 3 \Rightarrow 4 + \frac{3}{7}$

$$\frac{31}{7} = 4\frac{3}{7}$$

[2 marks]

Answer 2

(i)

$$4\frac{2}{5} = 4 + \frac{2}{5}$$

$$= \frac{4}{1} \times \mathbf{1} + \frac{2}{5} \qquad \text{Because } 4 = \frac{4}{1} \times 1$$

$$= \frac{4}{1} \times \frac{\mathbf{5}}{\mathbf{5}} + \frac{2}{5} \qquad \text{Lesson 1 technique}$$

$$= \frac{20}{5} + \frac{2}{5} \qquad \text{Lesson 1 technique}$$

$$= \frac{22}{5} \qquad \text{Lesson 1 technique}$$

[2 marks]

(ii)

$$9\frac{3}{4} = 9 + \frac{3}{4}$$

$$= \frac{9}{1} \times \mathbf{1} + \frac{3}{4} \qquad \text{Because } 9 = \frac{9}{1} \times 1$$

$$= \frac{9}{1} \times \frac{\mathbf{4}}{\mathbf{4}} + \frac{3}{4} \qquad \text{Lesson 1 technique}$$

$$= \frac{36}{4} + \frac{3}{4} \qquad \text{Lesson 1 technique}$$

$$= \frac{39}{4} \qquad \text{Lesson 1 technique}$$

[2 marks]

(iii)

$$50\frac{5}{6} = 50 + \frac{5}{6}$$

$$= \frac{50}{1} \times \mathbf{1} + \frac{5}{6} \qquad \text{Because } 50 = \frac{50}{1} \times 1$$

$$= \frac{50}{1} \times \frac{\mathbf{6}}{\mathbf{6}} + \frac{5}{6} \qquad \text{Lesson 1 technique}$$

$$= \frac{300}{6} + \frac{5}{6} \qquad \text{Lesson 1 technique}$$

$$= \frac{305}{6} \qquad \text{Lesson 1 technique}$$

[2 marks]

(iv)

$$108\frac{1}{3} = 108 + \frac{1}{3}$$

$$= \frac{108}{1} \times \mathbf{1} + \frac{1}{3} \qquad \text{Because } 108 = \frac{108}{1} \times 1$$

$$= \frac{108}{1} \times \frac{3}{3} + \frac{1}{3} \qquad \text{Lesson 1 technique}$$

$$= \frac{324}{3} + \frac{1}{3} \qquad \text{Lesson 1 technique}$$

$$= \frac{325}{3} \qquad \text{Lesson 1 technique}$$

[2 marks]

3.5.2 Solution to 3.4 Exercise

Answer 1

(i) $8\frac{3}{11} = \frac{8 \times 11 + 3}{11}$

$= \frac{88 + 3}{11}$

$= \frac{91}{11}$

(ii) $15\frac{2}{3} = \frac{15 \times 3 + 2}{3}$

$= \frac{45 + 2}{3}$

$= \frac{47}{3}$

(iii) $7\frac{9}{10} = \frac{7 \times 10 + 9}{10}$

$= \frac{70 + 9}{10}$

$= \frac{79}{10}$

(iv) $25\frac{3}{4} = \frac{25 \times 4 + 3}{4}$

$= \frac{100 + 3}{4}$

$= \frac{103}{4}$

[8 marks]

Answer 2

(i) {Multiples of 6} = {6, 12, **18**, 24, 30, 36, ...}

[1 mark]

(ii) {Multiples of 9} = {9, **18**, 27, 36, 45, 54 ...}

[1 mark]

(iii) *lcm*{6, 9} = 18

[1 mark]

(iv) $3\frac{5}{6} - 2\frac{5}{9} = \frac{3 \times 6 + 5}{6} - \frac{2 \times 9 + 5}{9}$

$= \frac{23}{6} \times \mathbf{1} - \frac{23}{9} \times \mathbf{1}$

$= \frac{23}{6} \times \frac{\mathbf{3}}{\mathbf{3}} - \frac{23}{9} \times \frac{\mathbf{2}}{\mathbf{2}}$ Use *lcm*{6,9} is 18

$= \frac{69}{18} - \frac{46}{18}$

$= \frac{23}{18}$

$= 1\frac{5}{18}$

[8 marks]

Answer 3

(i) {Multiples of 4} = {4, 8, 12, 16, **20**, 24, ...}

[1 mark]

(ii) {Multiples of 10} = {10, **20**, 30, 40, 50, 60, ...}

[1 mark]

(iii) $lcm\{4, 10\} = 20$

[1 mark]

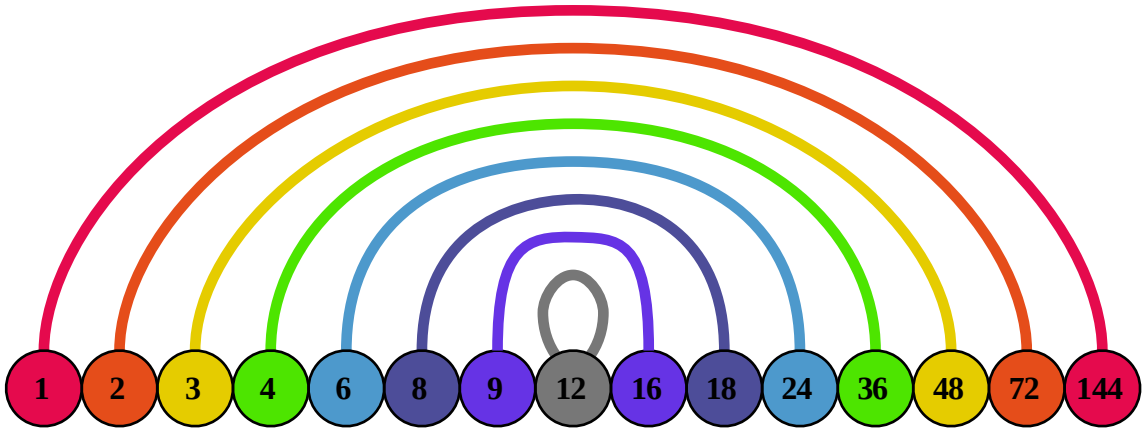
(iv)
$$5\frac{3}{4} + 3\frac{3}{10} = \frac{5 \times 4 + 3}{4} + \frac{3 \times 10 + 3}{10}$$
$$= \frac{23}{4} \times \mathbf{1} + \frac{33}{10} \times \mathbf{1}$$
$$= \frac{23}{4} \times \frac{\mathbf{5}}{\mathbf{5}} + \frac{33}{10} \times \frac{\mathbf{2}}{\mathbf{2}}$$ Use $lcm\{4,10\}$ is 20
$$= \frac{115}{20} + \frac{66}{20}$$
$$= \frac{181}{20}$$
$$= 9\frac{1}{20}$$

[8 marks]

Lesson 4

Year 6, Primary 6, Mathematics
Vulgar Fractions

Factor Rainbows



It's all about Hue

4.1 The factors of a number

The factors of a given number are all of the numbers that divide into it with zero remainder. The diagram shows the fifteen factors of 144.

As a list they are,

{Factors of 144} = {1, 2, 3, 4, 6, 8, 9, 12, 16, 18, 24, 36, 48, 72, 144}

Notice that the factors come in ***product pairs*** except the 12 in the middle.
(The word ***product*** means ***multiplication***)

The **product pairs** are,

$1 \times 144 = 144$ {1,144}

$2 \times 72 = 144$ {2,72}

$3 \times 48 = 144$ {3,48}

$4 \times 36 = 144$ {4,36}

$6 \times 24 = 144$ {6,24}

$8 \times 18 = 144$ [8,18}

$9 \times 16 = 144$ {9,16}

$12 \times 12 = 144$ {12}

The fact that each pair multiplies to give the 144 is a terrific help in not missing out any of the factors. Drawing the rainbow is a visual aid to remember this fact.

Question : Why is 5 NOT a factor of 144 ?

Answer #1 :

5 is not a factor of 144 because 144 does not divide by 5 exactly, there is a remainder. This can be shown by bus stop division.

$$\frac{144}{5} \Rightarrow 5\overline{)1\ 4\ 4} \Rightarrow \begin{array}{r} 0\ \ 2\ \ 8\ \ r\ \ 4 \\ 5\overline{)1\ {}^{1}4\ {}^{4}4} \end{array} \Rightarrow 28 + \frac{4}{5}$$

$$\frac{144}{5} = 28\frac{4}{5}$$

Answer #2 :

There is an alternative “clever” answer for this particular example.

Numbers that divide by 5 exactly end in a 0 or a 5.

As 144 ends in a 4 it does not divide by 5 exactly so 5 is not a factor of 144.

4.2 Exercise

Marks available : 40

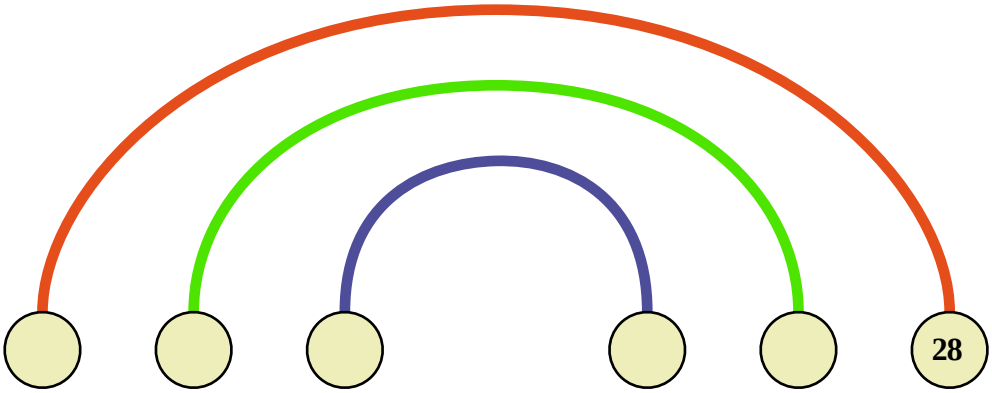
Question 1

Use bus stop division to show that 7 is not a factor of 144

[2 marks]

Question 2

(i) Complete the factor rainbow for 28



[2 marks]

(ii) Write in a list, all the factors of 28

[1 mark]

(iii) Use bus stop division to show that 3 is not a factor of 28

[2 marks]

Question 3

(i) Use bus stop division to show that 7 is not a factor of 75

[2 marks]

(ii) Without using bus stop division explain why 2 is not a factor of 75

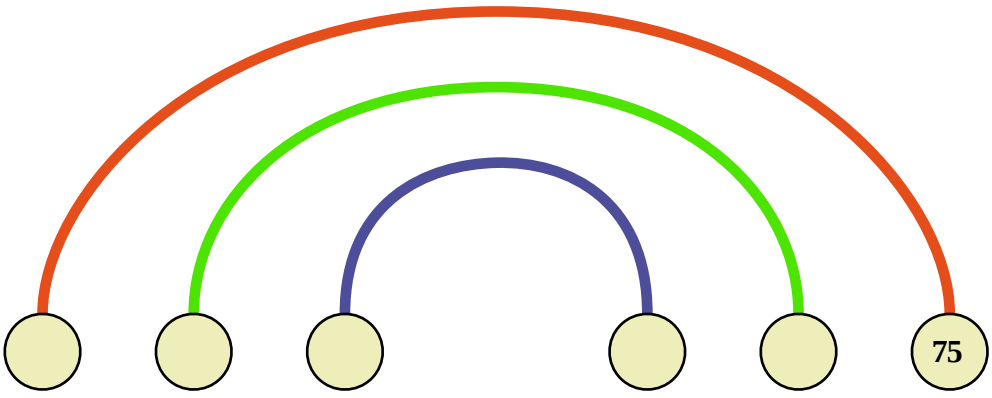
[2 marks]

(iii) As 2 is not a factor of 75, 4 is also not a factor of 75.
This is because 2 is a factor of 4.
(So if 4 was a factor so would be 2)
In fact, because 2 is not a factor, none of the following can be either;
{2, 4, 6, 8, 10, 12, 14, ...}

Write down a list of numbers that cannot be factors of 75 because of part (i) where you showed that 7 is not a factor.

[2 marks]

(iv) Complete the factor rainbow for 75



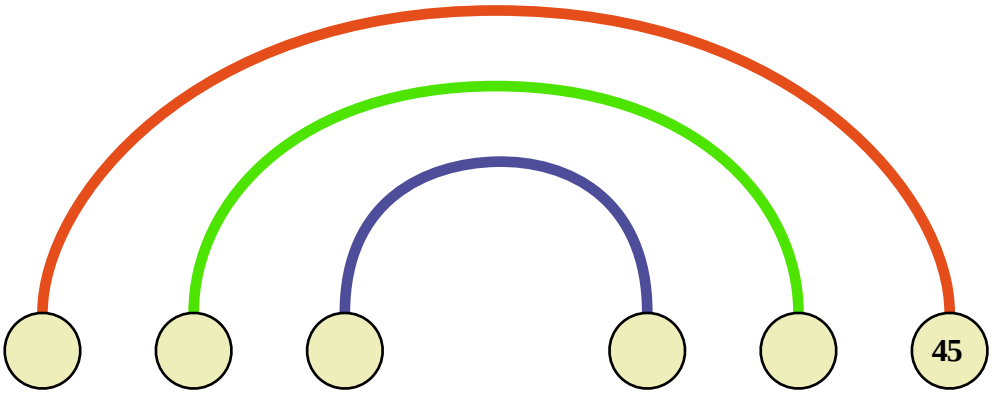
[2 marks]

(v) Write in a list, all the factors of 75

[1 mark]

Question 4

(i) Complete the factor rainbow for 45



[4 marks]

(ii) List all the factors of 45

[1 mark]

Question 5

(i) Without using bus stop division explain why 5 is not a factor of 56

[2 marks]

(ii) As 5 is not a factor of 56, list some related numbers also not factors.

[2 marks]

(iii) Without using bus stop division explain why 3 is not a factor of 56

[2 marks]

(iv) As 3 is not a factor of 56, list some related numbers also not factors.

[2 marks]

(v) Complete the factor rainbow for 56

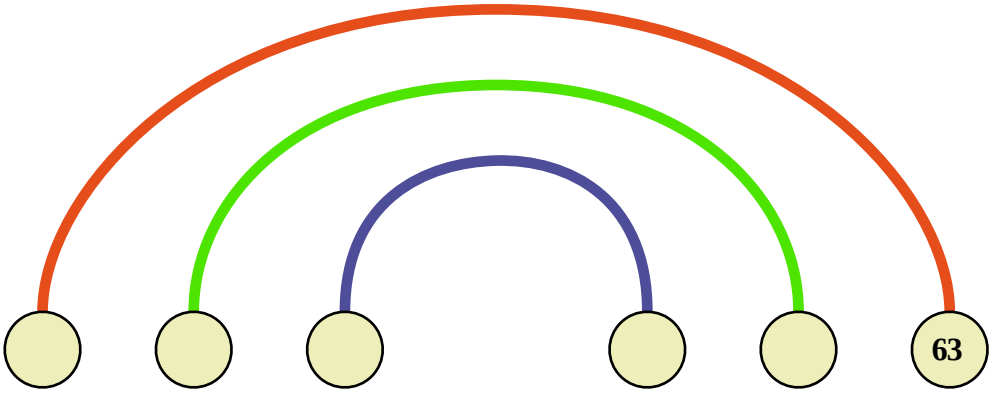
[4 marks]

(v) Write in a list, all the factors of 56

[1 mark]

Question 6

(i) Complete the factor rainbow for 63



[5 marks]

(ii) Write in a list, all the factors of 63

[1 mark]

4.3 Answers

4.3.1 Solutions to 4.2 Exercise

Answer 1

7 is not a factor of 144 because 144 does not divide by 7 exactly; there is a remainder. This can be shown by bus stop division.

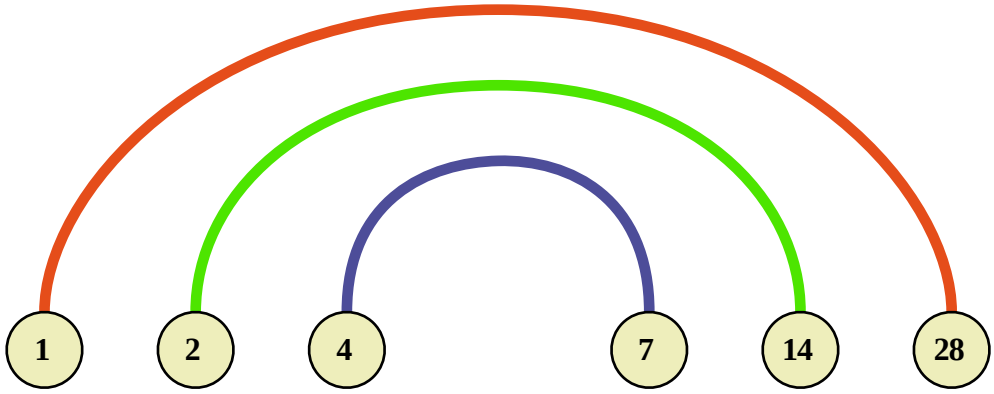
$$\frac{144}{7} \Rightarrow 7\overline{)1\ 4\ 4} \Rightarrow \begin{array}{r} 0\ 2\ 0\ r\ 4 \\ 7\overline{)1\ {}^{1}4\ 4} \end{array} \Rightarrow 20 + \frac{4}{7}$$

$$\frac{144}{7} = 20\frac{4}{5}$$

[2 marks]

Answer 2

(i)



[2 marks]

(ii) {Factors of 28} = {1, 2, 4, 7, 14, 28}

[1 mark]

(iii) 3 is not a factor of 28 because 28 does not divide by 3 exactly; there is a remainder. This can be shown by bus stop division.

$$\frac{28}{3} \Rightarrow 3\overline{)2\ 8} \Rightarrow \begin{array}{r} 0\ 9\ r\ 1 \\ 3\overline{)2\ {}^{2}8} \end{array} \Rightarrow 9 + \frac{1}{3}$$

$$\frac{28}{3} = 9\frac{1}{3}$$

[2 marks]

Answer 3

(i) 7 is not a factor of 75 because 75 does not divide by 7 exactly; there is a remainder. This can be shown by bus stop division.

$$\frac{75}{7} \Rightarrow 7\overline{)\,7\ 5} \Rightarrow \begin{array}{r} 1\ 0\ r\ 5 \\ 7\overline{)\,7\ 5} \end{array} \Rightarrow 10 + \frac{5}{7}$$

$$\frac{75}{7} = 10\frac{5}{7}$$

[2 marks]

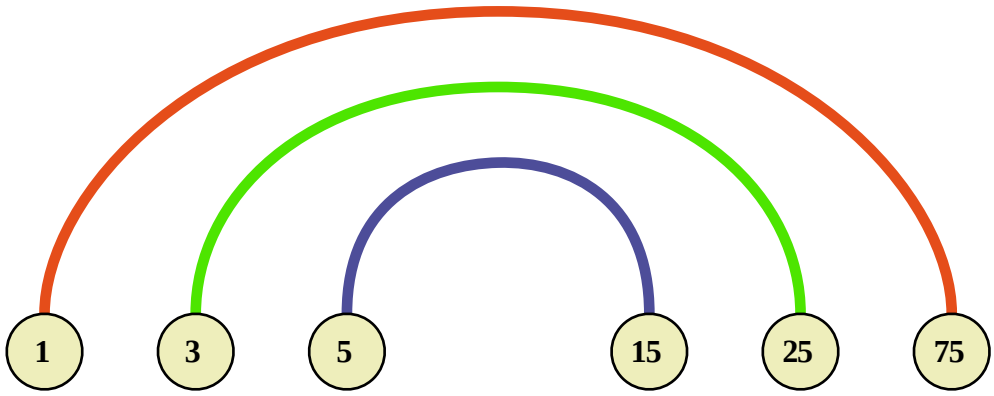
(ii) Numbers that divide by 2 exactly end in 0 or 2 or 4 or 6 or 8.
As 75 ends in a 5 it does not divide by 2 exactly so 2 is not a factor of 75.

[2 marks]

(iii) {Multiples of 7} = {7, 14, 21, 28, 35, 42, 49, 56, 63, 70, ...}

[2 marks]

(iv)



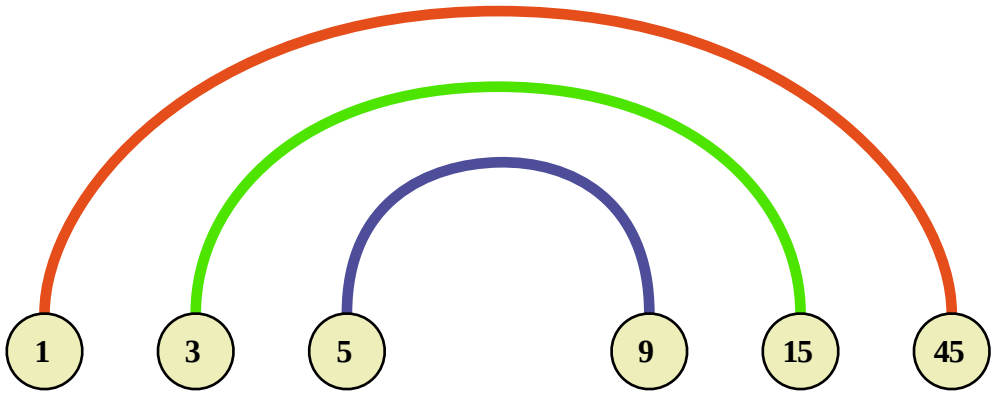
[2 marks]

(v) {Factors of 75} = {1, 3, 5, 15, 25, 75}

[1 mark]

Answer 4

(i)



[4 marks]

(ii) {Factors of 45} = {1, 3, 5, 9, 15, 45}

[1 mark]

Answer 5

(i) Numbers that divide by 5 exactly end in a 0 or a 5.
As 56 ends in a 6 it does not divide by 5 exactly so 5 is not a factor of 56.

[2 marks]

(ii) {Multiples of 5} = {5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, ... }

[2 marks]

(iii) Numbers that divide by 3 exactly have a sum of digits that divides by 3.
As 56 has a sum of digits given by,

$$5 + 6 \Rightarrow 11 \Rightarrow 1 + 1 = 2$$

and 2 does not divide by 3 nor does 56 so 3 is not a factor of 56.

[2 marks]

(iv) {Multiples of 3} = {3, 6, 9, 12, 15, 18, 21, 24, 27, ... }

[2 marks]

(v)

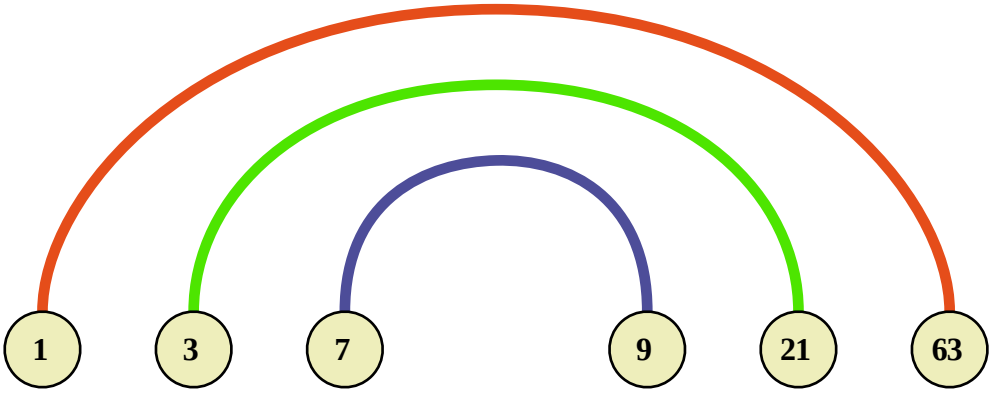
[4 marks]

(vi) {Factors of 56} = {1, 2, 4, 7, 8, 14, 28, 56}

[1 mark]

Answer 6

(i)



[5 marks]

(ii) {Factors of 63} = {1, 3, 7, 9, 21, 63}

[1 mark]

Lesson 5

Year 6, Primary 6, Mathematics
Vulgar Fractions

Prime Numbers

Count Von Count explains that 24 has 8 factors
{Factors of 24} = {1, 2, 3, 4, 6, 8, 12, 24}
He asks, "Is 24 a prime number ?"

5.1 How many factors ?

In the lesson on ***Factor Rainbows*** we saw that 144 has the factors,

{Factors of 144} ={1, 2, 3, 4, 6, 8, 9, 12, 16, 18, 24, 36, 48, 72, 144}

The curly brackets indicate that this is a ***set***; a collection of numbers.
We can count how many factors are in this set.

$$n\{\text{Factors of } 144\} = 15$$

Notice the *n* stands for "*n*umber of".

5.2 Exercise

Marks available : 30

Question 1

What is $n\{\text{Factors of } 6\}$?

[2 marks]

Question 2

What is $n\{\text{Factors of } 30\}$?

[2 marks]

Question 3

The table below lists all of the integers between 1 and 100 (in black), giving each its own square. Also in each square is a smaller number (in red). What is the smaller number telling you about the bigger number ?

[2 marks]

1 1	2 2	3 2	4 3	5 2	6 4	7 2	8 4	9 3	10 4
11 2	12 6	13 2	14 4	15 4	16 5	17 2	18 6	19 2	20 6
21 4	22 4	23 2	24 8	25 3	26 4	27 4	28 6	29 2	30 8
31 2	32 6	33 4	34 4	35 4	36 9	37 2	38 4	39 4	40 8
41 2	42 8	43 2	44 6	45 6	46 4	47 2	48 10	49 3	50 6
51 4	52 6	53 2	54 8	55 4	56 8	57 4	58 4	59 2	60 12
61 2	62 4	63 6	64 7	65 4	66 8	67 2	68 6	69 4	70 8
71 2	72 12	73 2	74 4	75 6	76 6	77 4	78 8	79 2	80 10
81 5	82 4	83 2	84 12	85 4	86 4	87 4	88 8	89 2	90 12
91 4	92 6	93 4	94 4	95 4	96 12	97 2	98 6	99 6	100 9

Question 4

Use the table to write down,

(i) $n\{\text{Factors of } 93\}$

[1 mark]

(ii) $n\{\text{Factors of } 52\}$

[1 mark]

(iii) $n\{\text{Factors of } 66\}$

[1 mark]

(iv) $n\{\text{Factors of } 97\}$

[1 mark]

Question 5

It turns out that numbers with exactly 2 factors are of extraordinary importance.
So much so that they are given a special name.
They are the ***prime*** numbers.

(i) On the table shade out (get rid of) all the numbers that are NOT prime.

[6 marks]

(ii) How many prime numbers are there less than 100 ?

[1 mark]

(iii) Explain why 91 is NOT a prime number.

[1 mark]

(iv) Explain why 89 is a prime number.

[1 mark]

Question 6

(i) Draw a factor rainbow for 84

[9 marks]

(ii) Write in a list, all the factors of 84

[1 mark]

(iii) Write in a list, all the ***prime*** factors of 84

[1 mark]

5.3 Answers

5.3.1 Solutions to 5.2 Exercise

Answer 1

{Factors of 6} = {1, 2, 3, 6}

So n(Factors of 6} = 4

[2 marks]

Answer 2

{Factors of 30} = {1, 2, 3, 5, 6, 10, 15, 30}

So n(Factors of 30} = 8

[2 marks]

Answer 3

The smaller (red) number gives the number of factors.

[2 marks]

Answer 4

(i) n(Factors of 93} = 4

[1 mark]

(ii) n(Factors of 52} = 6

[1 mark]

(iii) n(Factors of 66} = 8

[1 mark]

(iv) n(Factors of 97} = 2

[1 mark]

Answer 5

(i)

1 1	2 2	3 2	4 3	5 2	6 4	7 2	8 4	9 3	10 4
11 2	12 6	13 2	14 4	15 4	16 5	17 2	18 6	19 2	20 6
21 4	22 4	23 2	24 8	25 3	26 4	27 4	28 6	29 2	30 8
31 2	32 6	33 4	34 4	35 4	36 9	37 2	38 4	39 4	40 8
41 2	42 8	43 2	44 6	45 6	46 4	47 2	48 10	49 3	50 6
51 4	52 6	53 2	54 8	55 4	56 8	57 4	58 4	59 2	60 12
61 2	62 4	63 6	64 7	65 4	66 8	67 2	68 6	69 4	70 8
71 2	72 12	73 2	74 4	75 6	76 6	77 4	78 8	79 2	80 10
81 5	82 4	83 2	84 12	85 4	86 4	87 4	88 8	89 2	90 12
91 4	92 6	93 4	94 4	95 4	96 12	97 2	98 6	99 6	100 9

[6 marks]

(ii) There are 25 primes less than 100.

[1 mark]

(iii) A prime has 2 factors exactly.
91 has 4 factors, {1, 7, 13, 91}
So 91 is not prime.

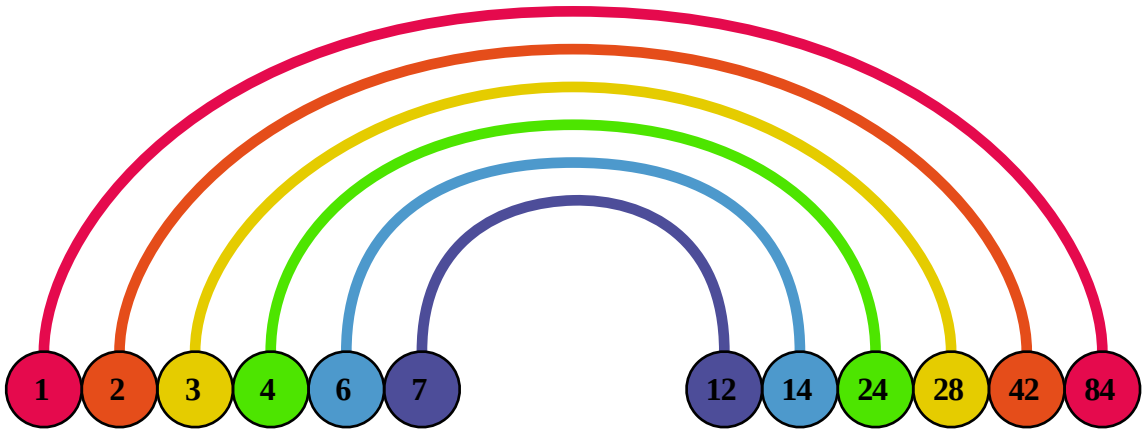
[1 mark]

(iv) A prime has 2 factors exactly.
89 has 2 factors, {1, 89}
So 89 is prime.

[1 mark]

Answer 6

(i)



[9 marks]

(ii) {Factors of 84} = {1, 2, 3, 4, 6, 7, 12, 14, 24, 28, 42, 84}

[1 mark]

(iii) {Prime factors of 84} = {2, 3, 7}

[1 mark]

Lesson 6

Year 6, Primary 6, Mathematics
Vulgar Fractions

Products of Primes

Prime Number Art
The Twenty-five primes less than 100

6.1 Six Prime Facts

- A prime number is a positive integer with two factors exactly.
- A positive integer with more than two factors is said to be ***composite***.
- The number 1 is neither prime nor composite.
- There are twenty-five primes less than 100.
- Primes are important because the composite numbers are built from primes.
- Primes are the atoms, the building blocks, of the mathematical universe.

6.2 The fundamental theorem of arithmetic

All composite numbers can be written as a product of primes.
The representation of any composite number in this way is essentially unique.

6.2.1 Examples

The numbers 50, 121 and 19669 are composite numbers.
This fundamental theorem of arithmetic tell us that they are built from prime numbers multiplied together.
Here are the decompositions of those numbers into products of primes.

(i) $12 = 2 \times 2 \times 3$

(ii) $121 = 11 \times 11$

(iii) $19669 = 13 \times 17 \times 89$

6.3 Exercise

Marks available : 40

Question 1

Write each of the following composite numbers as a product of prime numbers.

(i) $8 =$

[2 marks]

(ii) $10 =$

[2 marks]

(iii) $18 =$

[2 marks]

(iv) $50 =$

[2 marks]

(v) $99 =$

[2 marks]

(vi) $70 =$

[2 marks]

Question 2

Explain why the number 23 cannot be written as a product of primes.

[2 marks]

Question 3
The number 441 is not prime.
Prove this by finding a number other than 1 or 441 that divides into it.

[3 marks]

Question 4
Is 2557884 a prime number ?
Explain your answer.

[2 marks]

Question 5
Is 41087335 a prime number ?
Explain your answer.

[2 marks]

Question 6
The number 847 is not prime.
Prove this by finding a number other than 1 or 847 that divides into it.

[3 marks]

Question 7
I am thinking of an even number.
Could the number I am thinking of be prime ?
Explain your answer.

[3 marks]

Question 8

Make your own piece of prime number art, similar to that shown earlier.
You can cut numbers out of old newspapers and magazines.
Or, on a walk, take photographs of door numbers, price labels, car registration plates...
The space below is for you to construct your math-art.

[13 marks]

6.4 Answers

6.4.1 Solutions to 6.3 Exercise

Answer 1

(i) $8 = 2 \times 2 \times 2 \quad (= 2^3)$

(ii) $10 = 2 \times 5$

(iii) $18 = 2 \times 3 \times 3 \quad (= 2 \times 3^2)$

(iv) $50 = 2 \times 5 \times 5 \quad (= 2 \times 5^2)$

(v) $99 = 3 \times 3 \times 11 \quad (= 3^2 \times 11)$

(vi) $70 = 2 \times 5 \times 7$

[12 marks]

Answer 2

23 is already a prime number

Only composite numbers can be written as products of primes

[2 marks]

Answer 3

As 4 + 4 + 1 = 9 which is divisible by 3, it follows that 441 is divisible by 3

In fact, {Factors of 441} = {1, 3, 7, 9, 21, 49, 63, 147, 441} so any of those (other than 1 or 441) are suitable answers.

[3 marks]

Answer 4

No

As 2557884 is an even number it divides by 2.

(If it were prime it would only divide by itself and 1)

[2 marks]

Answer 5

No

As 41087335 has a last digit of 5 it divides by 5.

(If it were prime it would only divide by itself and 1)

[2 marks]

Answer 6

It's fairly straightforward to spot that 847 divides by 7.

In fact, {Factors 847} = {1, 7, 11, 77, 121, 847} so other answers are possible.

[3 marks]

Answer 7

Yes, the number 2 is both even and prime. (It's the only even prime number)

[3 marks]

Answer 8 Math-Art question. **[13 marks]**

Lesson 7

Year 6, Primary 6, Mathematics
Vulgar Fractions

Cherry Picking Method

What is 300 metres tall and made of jelly and custard with a cherry on top ?

~ The Trifle Tower ~

7.1 Grow Then Pick

(a) Use the cherry picking method to write as products of primes.

(i) 666 **(ii)** 900

[3, 3 marks]

(b) Hence simplify $\frac{666}{900}$

[4 marks]

7.2 Exercise

Marks available : 40

Question 1

(a) Use the cherry picking method to write as products of primes.

(i) 308 **(ii)** 560

[3, 3 marks]

(b) Hence simplify $\frac{308}{560}$

[4 marks]

Question 2

(a) Use the cherry picking method to write as products of primes.

(i) 675 **(ii)** 750

[3, 3 marks]

(b) Hence simplify $\frac{675}{750}$

[4 marks]

Question 3

(a) Use the cherry picking method to write as products of primes.

(i) 5775 **(ii)** 6050

[3, 3 marks]

(b) Hence simplify $\frac{5775}{6050}$

[4 marks]

Question 4

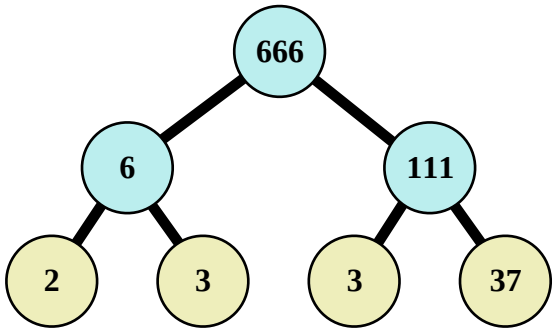
Write $\frac{2079}{1694}$ as a mixed number fraction in the form $a\frac{b}{c}$

[10 marks]

7.3 Answers

7.3.1 Solutions to 7.1 Grow Then Pick

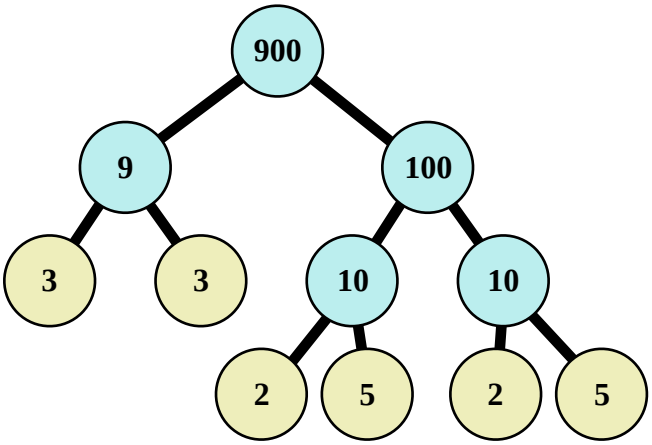
(a) **(i)**



$$666 = 2 \times 3 \times 3 \times 37$$

[3 marks]

(ii)



$$900 = 2 \times 2 \times 3 \times 3 \times 5 \times 5$$

[3 marks]

(b)

$$\frac{666}{900} = \frac{2 \times 3 \times 3 \times 37}{2 \times 2 \times 3 \times 3 \times 5 \times 5}$$

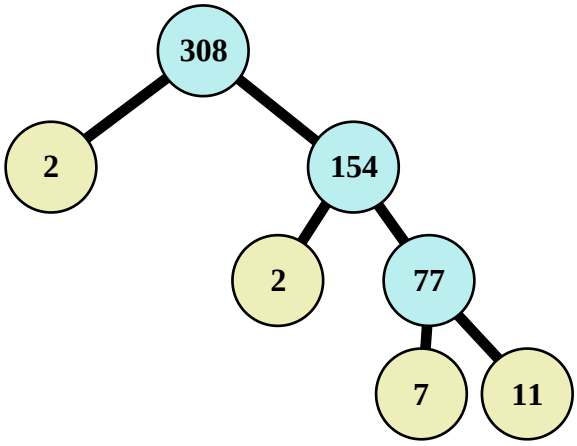
$$= \frac{37}{50}$$

[4 marks]

7.3.1 Solutions to 7.2 Exercise

Answer 1

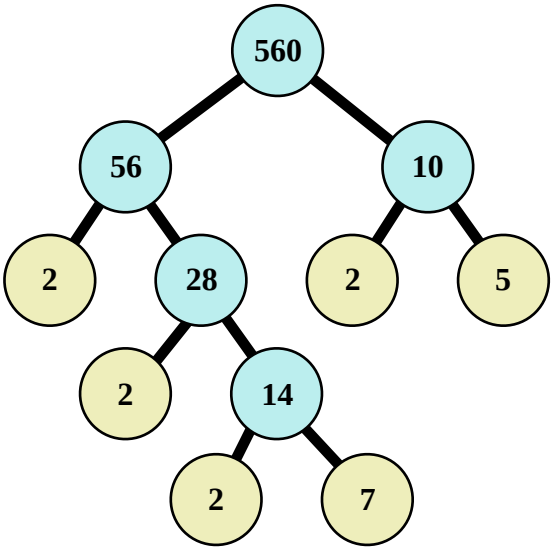
(a) **(i)**



$$308 = 2 \times 2 \times 7 \times 11$$

[3 marks]

(ii)



$$560 = 2 \times 2 \times 2 \times 2 \times 5 \times 7$$

[3 marks]

(b)

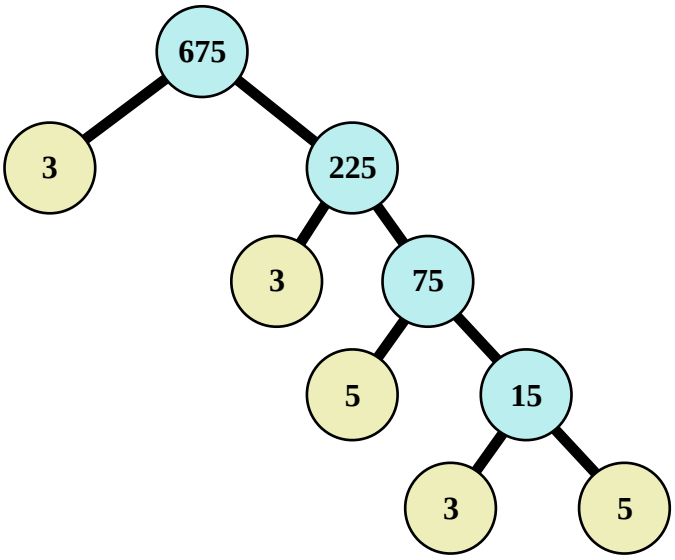
$$\frac{308}{560} = \frac{2 \times 2 \times 7 \times 11}{2 \times 2 \times 2 \times 2 \times 5 \times 7}$$

$$= \frac{11}{20}$$

[4 marks]

Answer 2

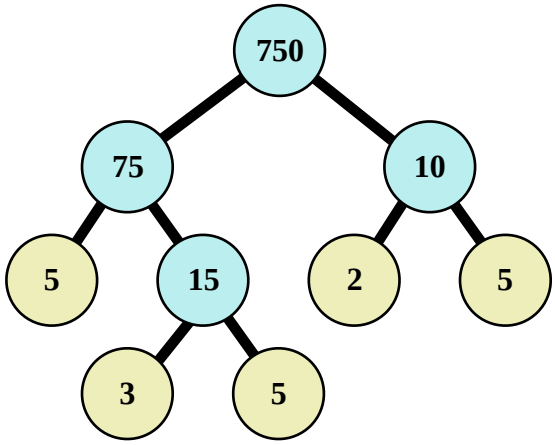
(a) **(i)**



$675 = 3 \times 3 \times 3 \times 5 \times 5$

[3 marks]

(ii)



$750 = 2 \times 3 \times 5 \times 5 \times 5$

[3 marks]

(b)

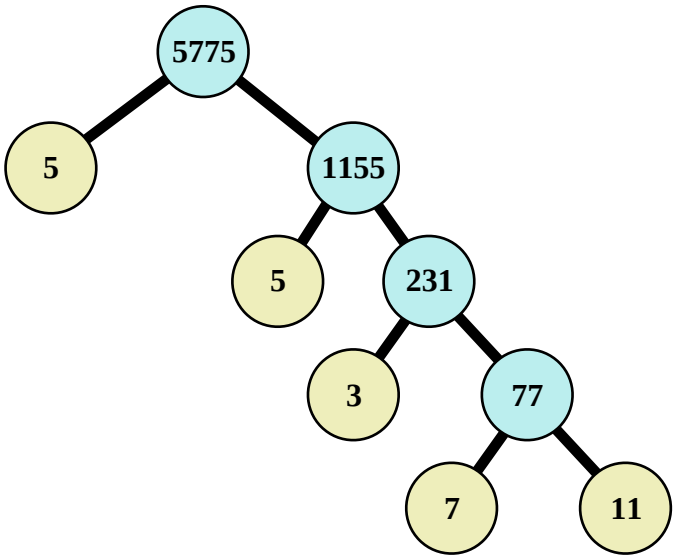
$$\frac{675}{750} = \frac{3 \times 3 \times 3 \times 5 \times 5}{2 \times 3 \times 5 \times 5 \times 5}$$

$$= \frac{9}{10}$$

[4 marks]

Answer 3

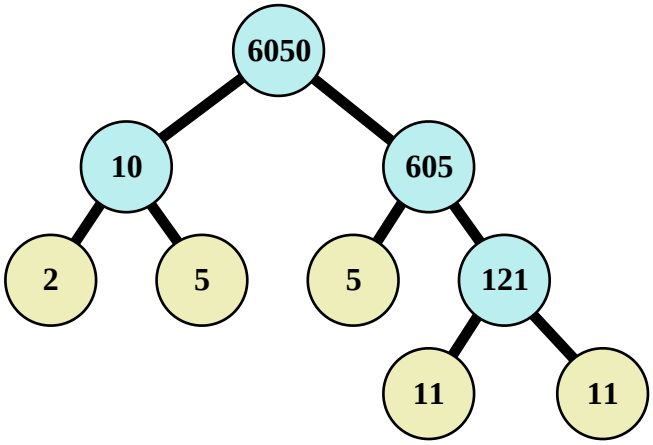
(a) **(i)**



$$5775 = 3 \times 5 \times 5 \times 7 \times 11$$

[3 marks]

(ii)



$$6050 = 2 \times 5 \times 5 \times 11 \times 11$$

[3 marks]

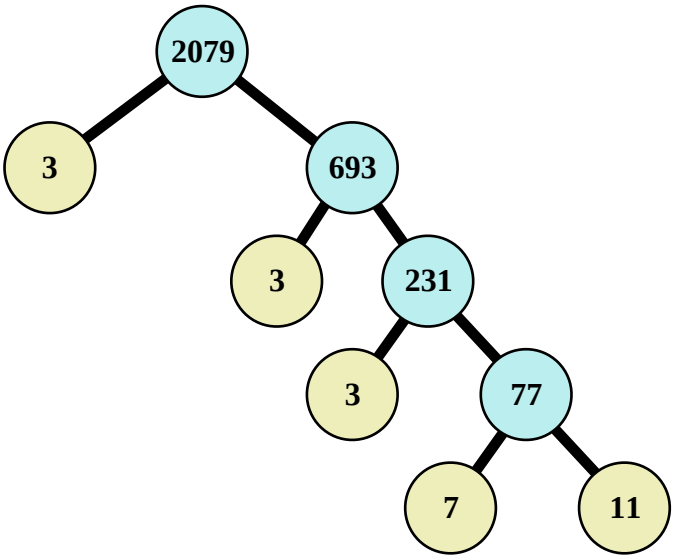
(b)

$$\frac{5775}{6050} = \frac{3 \times 5 \times 5 \times 7 \times 11}{2 \times 5 \times 5 \times 11 \times 11}$$

$$= \frac{21}{22}$$

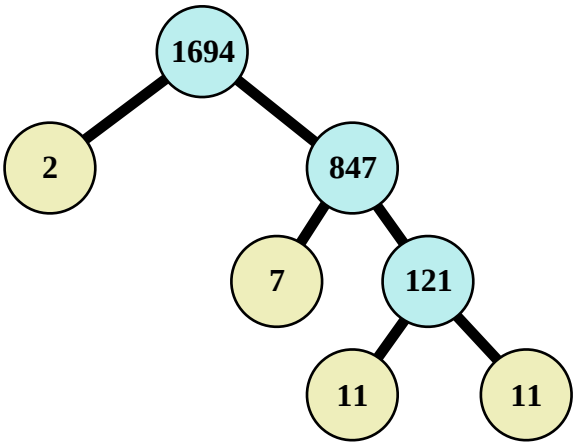
[4 marks]

Answer 4



$$2079 = 3 \times 3 \times 3 \times 7 \times 11$$

[3 marks]



$$1694 = 2 \times 7 \times 11 \times 11$$

[3 marks]

$$\frac{2079}{1694} = \frac{3 \times 3 \times 3 \times 7 \times 11}{2 \times 7 \times 11 \times 11}$$

$$= \frac{27}{22}$$

$$= 1\frac{5}{22}$$

[4 marks]

Lesson 8

Year 6, Primary 6, Mathematics
Vulgar Fractions

Test Revision

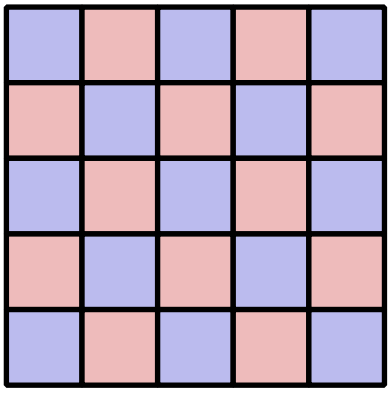
To show how good you are at fractions ...
... only do half your homework.

8.1 The Revision

Marks available : 50

Question 1

A slice of Battenberg cake is made of 25 equally sized square pieces, alternatively coloured lilac and pink, as shown.



Freddie says that if he eats all the lilac squares and George eats all the pink they will have eaten half of the slice each.

It Freddie correct ?

Explain your answer.

[3 marks]

Question 2

(i) List the first six multiples of 6

[1 mark]

(ii) List the first six multiples of 9

[1 mark]

(iii) What is *lcm*{6,9} ?

[1 mark]

(iv) Showing all the steps, work out $\frac{1}{6} + \frac{4}{9}$

[7 marks]

Question 3

Write the following top heavy fractions as mixed number fractions.

(i) $\frac{8}{3}$ **(ii)** $\frac{57}{5}$

(iii) $\frac{85}{7}$ **(iv)** $\frac{513}{4}$

[8 marks]

Question 4

(i) Without using bus stop division explain why 5 is not a factor of 154

[2 marks]

(ii) As 5 is not a factor of 154, list some related numbers also not factors.

[2 marks]

(iii) Without using bus stop division explain why 3 is not a factor of 154

[2 marks]

(iv) As 3 is not a factor of 154, list some related numbers also not factors.

[2 marks]

(v) Complete the factor rainbow for 154

[3 marks]

(v) Write in a list, all the factors of 154

[1 mark]

Question 5

(a) Use the cherry picking method to write as products of primes.

(i) 693 **(ii)** 945

[3, 3 marks]

(b) Hence simplify $\frac{693}{945}$

[3 marks]

Question 6

Write the following mixed number fractions as top heavy fractions.

(i) $5\frac{2}{7}$ **(ii)** $11\frac{3}{4}$

(iii) $53\frac{5}{7}$ **(iv)** $241\frac{1}{3}$

[8 marks]

8.2 Answers

8.2.1 Solutions to 8.2 The Revision

Answer 1

Freddie is not correct.

There are an odd number of squares, 25, which does not have 2 as a factor.

Freddie eats the 13 lilac squares, George the 12 pink.

[3 marks]

Answer 2

(i) {Multiples of 6} = {6, 12, 18, 24, 30, 36, ...]

[1 mark]

(ii) {Multiples of 9} = {9, 18, 27, 36, 45, 54, ...]

[1 mark]

(iii) $lcm\{6,9\} = 18$

[1 mark]

(iv)

$$\frac{1}{6} + \frac{4}{9} = \frac{1}{6} \times \mathbf{1} + \frac{4}{9} \times 1 \qquad \text{Step 1: Multiply by 1}$$

$$= \frac{1}{6} \times \frac{\mathbf{3}}{\mathbf{3}} + \frac{4}{9} \times \frac{\mathbf{2}}{\mathbf{2}} \qquad \text{Step 2: Use } lcm\{9,24\} \text{ is 72}$$

$$= \frac{3}{18} + \frac{8}{18} \qquad \text{Step 3: Do the two multiplications}$$

$$= \frac{11}{18} \qquad \text{Step 4: Do the easy addition}$$

[7 marks]

Answer 3

(i) $\frac{8}{3} \Rightarrow 3\overline{)8} \Rightarrow \begin{array}{r} 2\ r\ 2 \\ 3\overline{)8} \end{array} \Rightarrow 2\frac{2}{3}$

[2 marks]

(ii) $\frac{57}{5} \Rightarrow 5\overline{)57} \Rightarrow \begin{array}{r} 11\ r\ 2 \\ 5\overline{)57} \end{array} \Rightarrow 11\frac{2}{5}$

[2 marks]

(iii) $\frac{85}{7} \Rightarrow 7\overline{)85} \Rightarrow \begin{array}{r} 12\ r\ 1 \\ 7\overline{)8\,{}^{1}5} \end{array} \Rightarrow 12\frac{1}{7}$

[2 marks]

(iv) $\frac{513}{4} \Rightarrow 4\overline{)513} \Rightarrow \begin{array}{r} 128\ r\ 1 \\ 4\overline{)5\,{}^{1}1\,{}^{3}3} \end{array} \Rightarrow 128\frac{1}{4}$

[2 marks]

Answer 4

(i) Numbers that divide by 5 exactly end in a 0 or a 5.
As 154 ends in a 4 it does not divide by 5 exactly so 5 is not a factor of 154.

[2 marks]

(ii) {Multiples of 5} = {5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, ... }

[2 marks]

(iii) Numbers that divide by 3 exactly have a sum of digits that divides by 3.
As 154 has a sum of digits given by,

$$1 + 5 + 4 \Rightarrow 10 \Rightarrow 1 + 0 = 1$$

and 1 does not divide by 3 nor does 154 so 3 is not a factor of 154.

[2 marks]

(iv) {Multiples of 3} = {3, 6, 9, 12, 15, 18, 21, 24, 27, ... }

[2 marks]

(v)

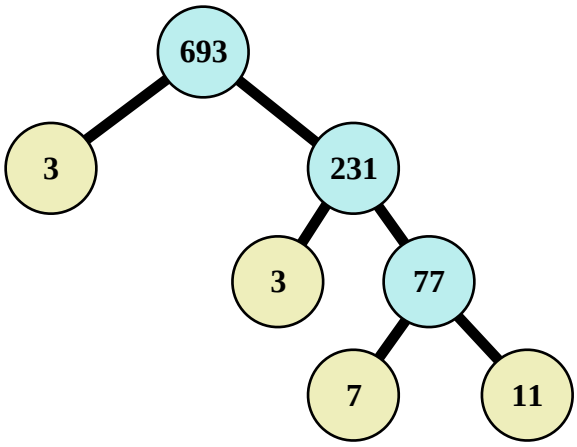
[3 marks]

(vi) {Factors of 154} = {1, 2, 7, 11, 14, 22, 77, 154}

[1 mark]

Answer 5

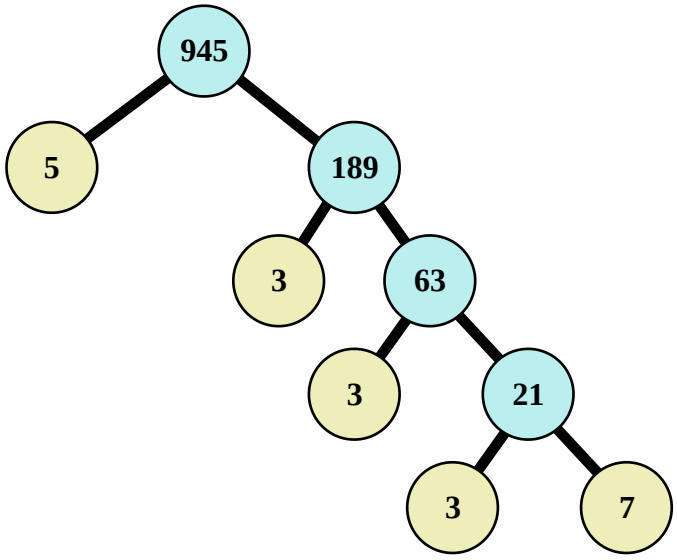
(a) **(i)**



$$693 = 3 \times 3 \times 7 \times 11$$

[3 marks]

(ii)



$$945 = 3 \times 3 \times 3 \times 5 \times 7$$

[3 marks]

(b)

$$\frac{693}{945} = \frac{3 \times 3 \times 7 \times 11}{3 \times 3 \times 3 \times 5 \times 7}$$

$$= \frac{11}{15}$$

[4 marks]

Answer 6

(i) $5\frac{2}{7} = \frac{5 \times 7 + 2}{7}$

$= \frac{35 + 2}{7}$

$= \frac{37}{7}$

(ii) $11\frac{3}{4} = \frac{11 \times 4 + 3}{4}$

$= \frac{44 + 3}{4}$

$= \frac{47}{4}$

(iii) $53\frac{5}{7} = \frac{53 \times 7 + 5}{7}$

$= \frac{60 + 5}{7}$

$= \frac{65}{7}$

(iv) $241\frac{1}{3} = \frac{241 \times 3 + 1}{3}$

$= \frac{723 + 1}{3}$

$= \frac{724}{3}$

[8 marks]

Lesson 9

Year 6, Primary 6, Mathematics
Vulgar Fractions

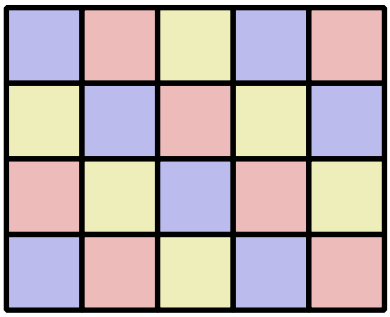
Test

9.1 The Test

Marks available : 50

Question 1

A slice of Battenberg cake of equally sized coloured squares is shown.



Annabella eats all the lilac, Frida all the pink, and Maddie all the yellow.
Did they each get a third of the slice ?
Explain your answer.

[3 marks]

Question 2

(i) List the first six multiples of 20

[1 mark]

(ii) List the first six multiples of 25

[1 mark]

(iii) What is *lcm*{20,25} ?

[1 mark]

(iv) Showing all the steps, work out $\frac{3}{20} + \frac{4}{25}$

[7 marks]

Question 3

Write the following top heavy fractions as mixed number fractions.

(i) $\frac{23}{4}$ **(ii)** $\frac{79}{7}$

(iii) $\frac{88}{5}$ **(iv)** $\frac{517}{5}$

[8 marks]

Question 4

(i) Without using bus stop division explain why 2 is not a factor of 275

[2 marks]

(ii) As 2 is not a factor of 275, list some related numbers also not factors.

[2 marks]

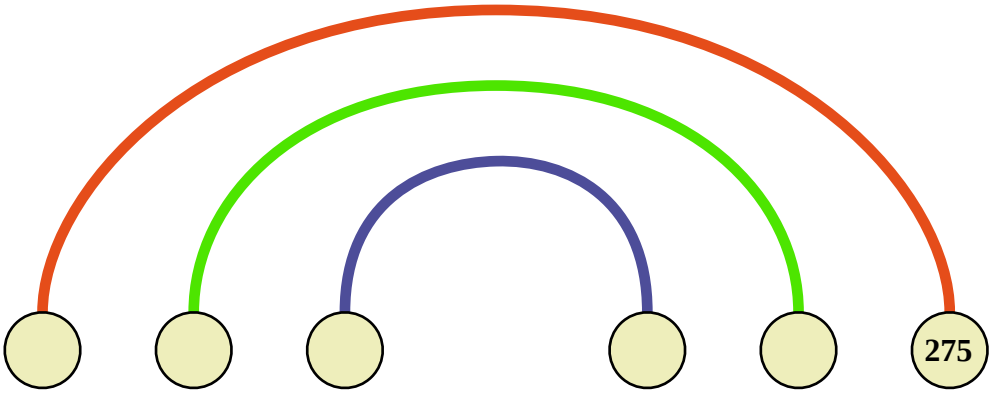
(iii) Without using bus stop division explain why 3 is not a factor of 275

[2 marks]

(iv) As 3 is not a factor of 275, list some related numbers also not factors.

[2 marks]

(v) Complete the factor rainbow for 275



[3 marks]

(v) Write in a list, all the factors of 275

[1 mark]

Question 5

(a) Use the cherry picking method to write as products of primes.

(i) 156 **(ii)** 650

[3, 3 marks]

(b) Hence simplify $\frac{156}{650}$

[3 marks]

Question 6

Write the following mixed number fractions as top heavy fractions.

(i) $6\frac{2}{3}$ **(ii)** $14\frac{3}{4}$

(iii) $71\frac{5}{7}$ **(iv)** $351\frac{1}{3}$

[8 marks]

9.2 Answers

9.2.1 Solutions to 9.2 The Test

Answer 1

They do not get a third each because there are 20 squares and 20 does not divide by 3.

Annabella eats 7 lilac squares, Frida 7 pink and Maddie 6 yellow.

[3 marks]

Answer 2

(i) {Multiples of 20} = {20, 40, 60, 80, 100, 120, ...]

[1 mark]

(ii) {Multiples of 25} = {25, 50, 75, 100, 125, 150, ...]

[1 mark]

(iii) $lcm\{20,25\} = 100$

[1 mark]

(iv)

$$\frac{3}{20} + \frac{4}{25} = \frac{3}{20} \times \mathbf{1} + \frac{4}{25} \times 1 \qquad \text{Step 1: Multiply by 1}$$

$$= \frac{3}{20} \times \frac{\mathbf{5}}{\mathbf{5}} + \frac{4}{25} \times \frac{\mathbf{4}}{\mathbf{4}} \qquad \text{Step 2: Use } lcm\{9,24\} \text{ is 72}$$

$$= \frac{15}{100} + \frac{16}{100} \qquad \text{Step 3: Do the two multiplications}$$

$$= \frac{31}{100} \qquad \text{Step 4: Do the easy addition}$$

[7 marks]

Answer 3

(i) $\frac{23}{4} \Rightarrow 4\overline{)2\ 3} \Rightarrow \begin{array}{r} 0\ 5\ r\ 3 \\ 4\overline{)2\ 3} \end{array} \Rightarrow 5\frac{3}{4}$

[2 marks]

(ii) $\frac{79}{7} \Rightarrow 7\overline{)7\ 9} \Rightarrow \begin{array}{r} 1\ 1\ r\ 2 \\ 7\overline{)7\ 9} \end{array} \Rightarrow 11\frac{2}{7}$

[2 marks]

(iii) $\frac{88}{5} \Rightarrow 5\overline{)8\ 8} \Rightarrow \begin{array}{r} 1\ 7\ r\ 3 \\ 5\overline{)8\ {}^{3}8} \end{array} \Rightarrow 17\frac{3}{5}$

[2 marks]

(iv) $\frac{517}{5} \Rightarrow 5\overline{)5\ 1\ 7} \Rightarrow \begin{array}{r} 1\ 0\ 3\ r\ 2 \\ 5\overline{)5\ 1\ {}^{1}7} \end{array} \Rightarrow 103\frac{2}{5}$

[2 marks]

Answer 4

(i) Numbers that divide by 2 exactly end in a 0 or 2 or 4 or 6 or 8.
As 275 ends in a 5 it does not divide by 2 exactly so 2 is not a factor of 275.

[2 marks]

(ii) {Multiples of 2} = {2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, ... }

[2 marks]

(iii) Numbers that divide by 3 exactly have a sum of digits that divides by 3.
As 275 has a sum of digits given by,

$$2 + 7 + 5 \Rightarrow 14 \Rightarrow 1 + 4 = 5$$

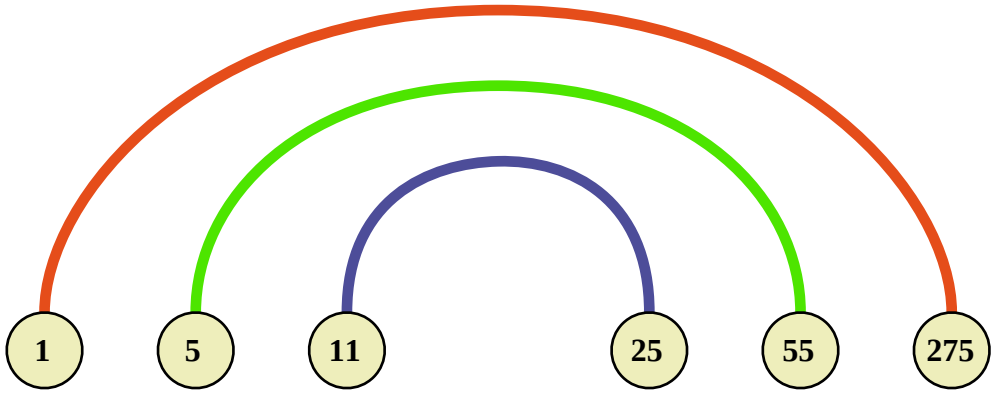
and 5 does not divide by 3 nor does 275 so 3 is not a factor of 275.

[2 marks]

(iv) {Multiples of 3} = {3, 6, 9, 12, 15, 18, 21, 24, 27, ... }

[2 marks]

(v)



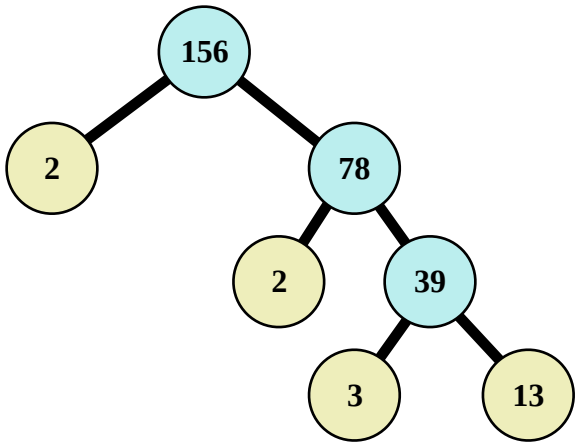
[3 marks]

(vi) {Factors of 275} = {1, 5, 11, 25, 55, 275}

[1 mark]

Answer 5

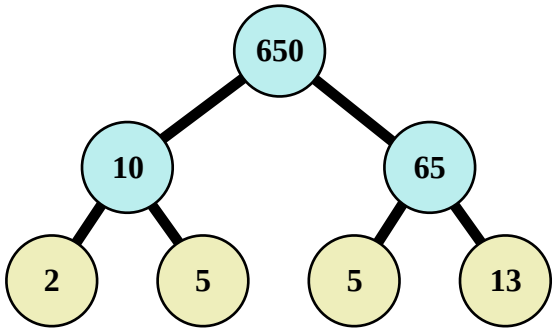
(a) **(i)**



$$156 = 2 \times 2 \times 3 \times 13$$

[3 marks]

(ii)



$$650 = 2 \times 5 \times 5 \times 13$$

[3 marks]

(b)

$$\frac{156}{650} = \frac{2 \times 2 \times 3 \times 13}{2 \times 5 \times 5 \times 13}$$

$$= \frac{6}{25}$$

[4 marks]

Answer 6

(i) $6\frac{2}{3} = \frac{6 \times 3 + 2}{3}$

$= \frac{18 + 2}{3}$

$= \frac{20}{3}$

(ii) $14\frac{3}{4} = \frac{14 \times 4 + 3}{4}$

$= \frac{56 + 3}{4}$

$= \frac{59}{4}$

(iii) $71\frac{5}{7} = \frac{71 \times 7 + 5}{7}$

$= \frac{497 + 5}{7}$

$= \frac{502}{7}$

(iv) $351\frac{1}{3} = \frac{351 \times 3 + 1}{3}$

$= \frac{1053 + 1}{3}$

$= \frac{1054}{3}$

[8 marks]