

1.8 Answers

1.8.1 Solutions to 1.4 Exercise

(i)	$6 + 9 \equiv x \pmod{12}$	(ii)	$10 + x \equiv 5 \pmod{12}$
	$x \equiv 15 \pmod{12}$		$x \equiv 5 - 10 \pmod{12}$
	$x \equiv 3 \pmod{12}$		$x \equiv -5 \pmod{12}$
			$x \equiv 7 \pmod{12}$
(iii)	$4 - 7 \equiv x \pmod{12}$	(iv)	$42 - x \equiv 8 \pmod{12}$
	$x \equiv -3 \pmod{12}$		$x \equiv 42 - 8 \pmod{12}$
	$x \equiv 9 \pmod{12}$		$x \equiv 34 \pmod{12}$
			$x \equiv 10 \pmod{12}$

[8 marks]

1.8.2 Solution to 1.6 Example (Teaching Video)

Note : This is a “beginners” method with a premium placed on “understanding”

$$\begin{aligned} 3x + 14 &\equiv 5 \pmod{6} \\ 3x &\equiv 5 - 14 \pmod{6} \\ 3x &\equiv -9 \pmod{6} \\ 3x &\equiv 3, 9, 15, 21, 27, \dots \pmod{6} \\ x &\equiv 1, 3, 5 \pmod{6} \end{aligned}$$

This could be more succinctly written as $x \equiv 1 \pmod{2}$

[6 marks]

1.8.3 Solutions to 1.7 Exercise

Answer 1

(i)	$29 \equiv 5 \pmod{12}$
(ii)	$-7 \equiv 5 \pmod{12}$
(iii)	$145 \equiv 1 \pmod{12}$
(iv)	$47 + 21 \equiv 68 \equiv 8 \pmod{12}$
(iv)	$3 - 19 \equiv -16 \equiv 8 \pmod{12}$
(vi)	$5^3 \equiv 125 \equiv 5 \pmod{12}$

[4 marks]

Answer 2**(a)** Least residues modulo 4 = { 0, 1, 2, 3 }**[1 mark]**

(b) (i) $7^0 \equiv 1 \pmod{4}$

(ii) $7^1 \equiv 7 \equiv 3 \pmod{4}$

(iii) $7^2 \equiv 49 \equiv 1 \pmod{4}$

(iv) $7^3 \equiv 343 \equiv 3 \pmod{4}$

(v) $7^4 \equiv 2401 \equiv 1 \pmod{4}$

(vi) $7^5 \equiv 16807 \equiv 3 \pmod{4}$

[3 marks]**(c)** From the pattern observed in part (b) it would seem that,

$$7^{EVEN} \equiv 1 \pmod{4}$$

$$7^{ODD} \equiv 3 \pmod{4}$$

(i) $7^{758} \equiv 1 \pmod{4}$

(ii) $7^{433} \equiv 3 \pmod{4}$

(iii) $7^{501} + 7^{383} \equiv 3 + 3 \equiv 6 \equiv 2 \pmod{4}$

[3 marks]**Answer 3**

(i) $6 + 4 \equiv x \pmod{8}$

$$x \equiv 10 \pmod{8}$$

$$x \equiv 2 \pmod{8}$$

(ii) $3 + x \equiv 1 \pmod{8}$

$$x \equiv 1 - 3 \pmod{8}$$

$$x \equiv -2 \pmod{8}$$

$$x \equiv 6 \pmod{8}$$

(iii) $4 - 19 \equiv x \pmod{8}$ **(iv)** $37 - x \equiv 8 \pmod{8}$

$$x \equiv -15 \pmod{8}$$

$$x \equiv 1 \pmod{8}$$

$$x \equiv 37 - 8 \pmod{8}$$

$$x \equiv 29 \pmod{8}$$

$$x \equiv 5 \pmod{8}$$

[8 marks]

Answer 4

$$2x + 11 \equiv 1 \pmod{6}$$

$$2x \equiv 1 - 11 \pmod{6}$$

$$2x \equiv -10 \pmod{6}$$

$$2x \equiv 2, 8, 14, \dots \pmod{6}$$

$$x \equiv 1, 4, 7, \dots \pmod{6}$$

$$x \equiv 1, 4 \pmod{6}$$

This could be more succinctly written as $x \equiv 1 \pmod{3}$

[6 marks]

Answer 5

$\times_5$	0	1	2	3	4
0	0	0	0	0	0
1	0	1	2	3	4
2	0	2	4	1	3
3	0	3	1	4	2
4	0	4	3	2	1

[4 marks]

Answer 6

- (a) (i) $1! \equiv 1 \pmod{7}$
(ii) $2! \equiv 2 \pmod{7}$
(iii) $3! \equiv 6 \pmod{7}$
(iv) $4! \equiv 24 \equiv 3 \pmod{7}$
(v) $5! \equiv 120 \equiv 1 \pmod{7}$
(vi) $6! \equiv 720 \equiv 6 \pmod{7}$
(vii) $7! \equiv 5040 \equiv 0 \pmod{7}$
(viii) $8! \equiv 40320 \equiv 0 \pmod{7}$
(ix) $9! \equiv 362880 \equiv 0 \pmod{7}$

[3 marks]

- (b) For $n \geq 7$, $n = 1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 \times \dots \times n$

As it contains a 7, it will divide by 7 and so be congruent to 0 modulo 7

[2 marks]

Answer 7

(a)	(i)	$13 \equiv 4 \pmod{9}$	$1 + 3 = 4$
	(ii)	$572 \equiv 5 \pmod{9}$	$5 + 7 + 2 = 14 \Rightarrow 1 + 4 = 5$
	(iii)	$334 \equiv 1 \pmod{9}$	$3 + 3 + 4 = 10 \Rightarrow 1 + 0 = 1$
	(iv)	$3102 \equiv 6 \pmod{9}$	$3 + 1 + 0 + 2 = 6$
	(v)	$1002001 \equiv 4 \pmod{9}$	$1 + 0 + 0 + 2 + 0 + 0 + 1 = 4$
	(vi)	$9994999 \equiv 4 \pmod{9}$	$9 + 9 + 9 + 4 + 9 + 9 + 9 = 58$ $\Rightarrow 5 + 8 = 13$ $\Rightarrow 1 + 3 = 4$

[3 marks]

(b) The answers can be obtained by summing the digits modulo 9, as shown above for the part (a) questions.

$$\begin{aligned}
 &12345678987654321 \\
 \Rightarrow &2(1 + 2 + 3 + 4 + 5 + 6 + 7 + 8) + 9 \\
 &= 2((1 + 8) + (2 + 7) + (3 + 6) + (4 + 5)) + 9 \\
 &= 2 \times 4 \times 9 + 9 \\
 &\equiv 0 \pmod{9}
 \end{aligned}$$

[2 marks]**Answer 8**

When $n \geq 5$, $n! \equiv 0 \pmod{15}$ because $n!$ then contains factors of both 3 and 5 and so contains a factor of 15.

$$\begin{aligned}
 &1! + 2! + 3! + 4! + \dots + 100! && \pmod{15} \\
 \equiv &1! + 2! + 3! + 4! + 0 + 0 + 0 + \dots + 0 && \pmod{15} \\
 \equiv &1 + 2 + 6 + 24 && \pmod{15} \\
 \equiv &3 && \pmod{15}
 \end{aligned}$$

So the remainder is 3

[5 marks]

Answer 9**(i)** For 51177875501

$$5 + 1 + 1 + 7 + 7 + 8 + 7 + 5 + 5 + 0 = 46$$

$$\Rightarrow 4 + 6 = 10$$

$$\Rightarrow 1 + 0 = 1$$

As 1 is the check digit, this serial number passes the security check.

(ii) For 88100245327

$$8 + 8 + 1 + 0 + 0 + 2 + 4 + 5 + 3 + 2 = 33$$

$$\Rightarrow 3 + 3 = 6$$

As 6 is not the check digit, this serial number does not pass.

[2 marks]**Answer 10**

$$1^2 \equiv 1 \pmod{4}$$

$$2^2 \equiv 4 \equiv 0 \pmod{4}$$

$$3^2 \equiv 9 \equiv 1 \pmod{4}$$

$$4^2 \equiv 16 \equiv 0 \pmod{4}$$

$$5^2 \equiv 25 \equiv 1 \pmod{4}$$

$$6^2 \equiv 36 \equiv 0 \pmod{4}$$

...

To prove the pattern suggested by the above, show that any odd number squared is congruent to 1 modulo 4 and that any even number squared is congruent to 0 modulo 4, which is not difficult.

$$\begin{aligned} & 1^2 + 2^2 + 3^2 + 4^2 + 5^2 + 6^2 + \dots + 100^2 \pmod{4} \\ & \equiv 1 + 0 + 1 + 0 + 1 + 0 + \dots + 0 \pmod{4} \\ & \equiv 50 \pmod{4} \\ & \equiv 2 \pmod{4} \end{aligned}$$

[4 marks]