SHREWSBURY SCHOOL.

MATHEMATICS PRIZE, 1953

1. Find values of x and n which satisfy the simultaneous equations

$$\left.\begin{aligned} x^{2n+2} - 9x^{n+2} - 4x^n + 36 = 0 \\ x^{2n-2} - 10x^{n-1} + 25 = 0 \end{aligned}\right\}$$

2. Justify the rule that, if the sum of the digits of a number is divisible by 3 or 9, then the number is divisble by 3 or 9 respectively.

Devise and prove a test for division by 11.

3. Solve the simultaneous equations

$$\left.\begin{aligned} x^2 - yz = a^2 \\ y^2 - zx = b^2 \\ z^2 - xy = c^2 \end{aligned}\right\}$$

4. An aircraft is to intercept a ship in the shortest possible time. The ship is 400 nautical miles due west of the aircraft, steaming at 30 knots on a heading S.20°E, and is subject to an ocean current of 10 knots towards the south-east. The aircraft cruises at 200 knots in still air ; the wind is blowing from N.70°W at a speed of 50 knots. *Describe*, with the aid of a diagram, a construction for finding the direction in which the aircraft must steer.

5. Show that there is no x^6 term in the product

$$(1-x^2)^3(1-x+x^2)(1+x+x^2+x^3)(1-x+x^2-x^3+x^4)(1+x+x^2+x^3+x^4+x^5)$$

6. A and B are two fixed points on a given circle, and P and Q are the extremities of any diameter of the same circle. Prove that the locus of the intersections of AP and BQ, and of AQ and BP, is the circle which cuts the given circle orthogonally (at right angles) at A and B.

[P.T.O.

7. Given that ten letters can be placed in ten addressed envelopes, so that every letter is in a wrong envleope in N_{10} different ways, where

$$N_{10} = 10! \left[\frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \frac{1}{5!} + \ldots\ldots + \frac{1}{10!} \right]$$

compute the value of N_{10}.

It takes a minute to place ten letters in ten envelopes, and a minute to take them all out again. If today you start putting the letters into the envelopes, every letter into a wrong envelope, taking them all out again and then reinserting them, find the date on which you will finish the task of inserting the letters in all the N_{10} different ways, if you work at the rate of eight hours a day for seven days a week.

8. A sum of £5. 0s. 6d. is made up of 27 coins which are sovereigns, half-crowns or shillings. How many coins are there of each sort ?

9. Show that the product of the Lowest Common Multiple and the Highest Common Factor of any *two* numbers is equal to the product of the numbers.

H denotes the Highest Common Factor, and L the Lowest Common Multiple, of *three* numbers. H_1, H_2, H_3 are the Highest Common Factors of the same numbers taken in pairs; L_1, L_2, L_3 are the corresponding Lowest Common Multiples. Show that

$$\frac{L_1 \; L_2 \; L_3}{H_1 \; H_2 \; H_3} = \frac{L^2}{H^2}$$

10. Explain how to construct a triangle when given

(i) the lengths of two sides and the median to the third side,

(ii) the lengths of the three medians.