Shrewsbury School

MATHEMATICS PRIZE, 1960

1. (*i*) If $a+b+c=0$, find the numerical value of

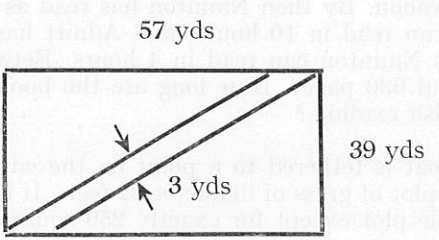
$$\frac{(a+b)^2+(b+c)^2+(c+a)^2}{(a+b)(b+c)+(b+c)(c+a)+(c+a)(a+b)}$$

(*ii*) Find the cube root of

$$(3x-y)^3+(x+2y)^3+3(4x+y)(3x^2+5xy-2y^2)$$

2. A man has a rectangular garden 57 yards long by 39 yards wide, and he makes a diagonal path, 3 yards wide, from corner to corner as shown in the diagram. What is the area of the path ?

(N.B.—You are advised not to use tables for this question)



3. A is the centre of a circle and B is any point outside it. BC is a straight line drawn from B to cut the circle at C. BD, BE are the tangents from B to the circle. Through C a straight line is drawn perpendicular to BC to cut AD in T and AE in U. Prove that $A\hat{T}B=A\hat{B}U$.

4. There are two factors, of which one is a quadratic, of the following expression. Find them.

$$9x^5-18x^4+20x^3-42x^2+39x-7.$$

5. (*i*) Find a number of four digits such that it is reversed when it is multiplied by 4.

(*ii*) In this multiplication sum the a's represent one digit, the b's another, and the asterisks missing digits. Rewrite it with the correct digits.

$$\begin{array}{cccccc} & 3 & * & a & b & \\ & & & * & b & \\ \hline * & * & * & * & & \\ * & 6 & * & * & a & \\ \hline * & * & * & * & 9 & \end{array}$$

6. "How old are you, John ?" I asked.

"I can never remember the answer to that," he replied.

"Surely you must have some idea ?" I said, for it seemed so stupid that he shouldn't even know his age.

"Oh, yes," he said, "this may help you to work it out : my brother is two years older than I, my sister is four years older than he, my mother says she was only 24 when I was born, and that when she got married my father was four years older than she was. My brother, who is a mathematical type rather like yourself, tells me that if I live to see the turn of the century, my age will be half as much again as the average age of our family will be in 1964. I can't say I care, but if he's right it may help you to work out my age, as you seem so inquisitive."

How old is John now ? And, if his father lives, when will he be 100 ?

7. Adnitt and Naunton are passionate book readers. Adnitt starts at 8 a.m. and Naunton at 11 a.m. to read books non-stop. They read steadily and finish at the same moment in the afternoon. By then Naunton has read as many pages as Adnitt can read in 10 hours, and Adnitt has read as many pages as Naunton can read in 4 hours. Between them they have read 630 pages. How long are the books and when do they finish reading ?

8. A goat is tethered to a point on the circumference of a circular plot of grass of diameter 32 feet. If it can graze over the whole plot except for exactly 256 square feet, how long is the tether ?

9. Make a full-scale drawing of this swastika on squared paper. Its overall dimensions are 7 ins. by 7 ins., the width is everywhere 1 in., and the feet are 2 ins. long. Show clearly on your drawing how it can be divided into four pieces which will fit together to form a perfect square.