THE ARNOLD HAGGER MATHEMATICS PRIZE

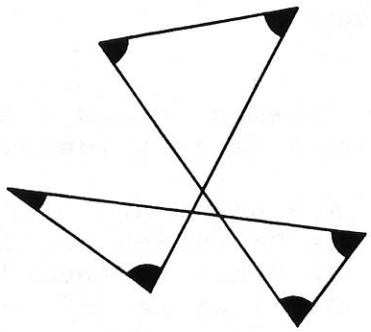
Wednesday, 21st February 1990 : 7.15 - 8.45

You may attempt as many questions as you like, in any order. The marks for each question are shown. Be careful to make your methods clear by including all working and reasoning: answers alone are not sufficient.

Calculators and tables may not be used.

1) A calls 1, B 7, A 12, B 22, A 23, and so on. Each call is higher by any number from 1 to 10. Whoever calls 100 wins. How does A win? [3]

2) For the figure shown calculate the sum of the marked angles.



[3]

3) Every day at noon a ship leaves Southampton for New York, and another ship leaves New York for Southampton. The trip lasts seven days and seven nights. How many New York - Southampton ships will the ship leaving Southampton today meet on its journey to New York? [3]

4) Two cups of tea, three cups of coffee and four cups of cocoa cost altogether less than £1; three cups of tea, four cups of coffee and two cups of cocoa cost altogether more than £1.

Will four cups of tea and five cups of coffee cost altogether more or less than £1?

Will one cup of coffee and eight cups of cocoa cost altogether less or more than £1?

Which will cost more: two cups of cocoa, or a cup of tea and a cup of coffee? [6]

5) A number is said to be palindromic if it reads the same backwards as forwards. For example, 7447 is palindromic. Show that all 4-digit palindromic numbers are divisible by eleven. [5]

6) Fred goes exploring in an underground tunnel which has a level floor and a semicircular cross-section of radius 3.4 metres. He finds a large chest, 1.6 metres high, and with a square base of side 1.8 metres. Unfortunately, the edge of the lid opposite the hinge is jammed hard against the wall of the tunnel. Suspecting that the chest contains land-rover parts (well I had to get it in somewhere, didn't I?), Fred decides to saw the lid in two, parallel to the hinged edge, to obtain a rectangular flap. What is the area of the largest flap which Fred can obtain if he wants the flap to fold right back? [5]

7) How many zeros are there at the end of the number which is the product of the first two hundred positive integers? [5]

8) A polygon, not necessarily regular, is drawn so that its sides touch a given circle of radius r. The perimeter of the polygon has length P. Find the area of the polygon in terms of r and P. [5]

9) Frogs always speak the truth and toads always tell lies. From what is said identify A,B,C and P,Q,R as frogs or toads.

(i) A,B and C are holding a discussion:
A: "We three are frogs."
B: "That is true."
C: "That is not true."

(ii) Now P,Q and R are meeting:
P: "We three are toads."
Q: "Exactly one of us is a frog."
R remains silent. [10]

10) In the addition sum below the different letters used denote different digits. Find which digit is represented by each letter, showing that you have obtained the only possible solution

$$\begin{array}{r} F\ O\ R\ T\ Y \\ \quad\ T\ E\ N \\ \quad\ T\ E\ N \\ \hline S\ I\ X\ T\ Y \end{array}$$

[10]

11) Suppose that you walk slowly down a downward-moving escalator and take 50 steps of the escalator to reach the bottom; then by running five times as fast up the same excalator you reach the top after taking 125 steps of the escalator. Assuming constant speeds for walking, running and for the escalator, find how many steps the escalator shows when it is stationary. [10]

12) Calculate the squares of: 67, 667, 6667, and 66667.

Guess the value of the square of the number consisting of one million sixes followed by a seven. Now prove it. [15]