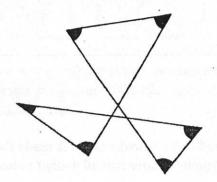# THE

ARNOLD HAGGER

MATHEMATICS

PRIZE

Wednesday, 23rd January 2002

$1\frac{1}{2}$ hours

- *Answer as many questions as you can, in any order, on the penal or graph penal provided. There are 14 questions on this paper and the marks for each question are shown in brackets [ ]. Do not expect to finish the paper: you are doing well if you finish only a few of the questions.*

- *Your work will be marked on the elegance and mathematical coherence of the solutions rather than the answers alone. Most of the marks awarded will be for clear and concise reasoning.*

- *Calculators may not be used.*

1.   Tweedledum throws an ordinary dice, then Tweedledee throws the same dice. What is the probability that Tweedledum throws a higher number than Tweedledee?   [5]

2.   A reservoir is in the form of a perfect circle. A perch (as opposed to a rod or pole: this question is in metric units) starts at the edge and swims 600m due north before meeting the edge again. The fish then turns due east and swims a further 800m before running aground. What is the diameter of the reservoir?   [5]

3.   For the figure shown calculate the sum of the marked angles.

                    [5]

4.   Frogs always speak the truth: toads always tell lies. From what is said identify A,B,C and P,Q,R as frogs or toads.

     (i)   A, B and C are holding a discussion:
                    A: "We three are frogs."
                    B: "That is true."
                    C: "That is not true."   [5]

     (ii)   Now P, Q and R are meeting:
                    P: "We three are toads."
                    Q: "Exactly one of us is a frog."
                    R remains silent   [5]

5.   Every day at noon a ship leaves Southampton for New York, and another ship leaves New York for Southampton. The trip lasts seven days and seven nights. How many New York to Southampton ships will the ship leaving Southampton today meet at sea on its journey to New York?   [5]

6.   Two cups of tea, three cups of coffee and four cups of cocoa cost altogether less than £5. Three cups of tea, four cups of coffee and two cups of cocoa cost altogether more than £5.

     (a)   Will four cups of tea and five cups of coffee cost altogether more or less than £5?

     (b)   Will one cup of coffee and eight cups of cocoa cost altogether more or less than £5?

     (c)   Which will cost more: two cups of cocoa, or a cup of tea and a cup of coffee?
                    [5]

TURN OVER

7. In the addition sum below the different letters used denote different digits.

$$\begin{array}{r} F\ O\ R\ T\ Y \\ T\ E\ N \\ \underline{T\ E\ N} \\ S\ I\ X\ T\ Y \end{array}$$

Find which digit is represented by each letter,   [5]
showing that you have obtained the only possible solution.   [5]

8. Alan and Bob are playing a counting game. Alan starts by calling a number from 1 to 10,
then Bob calls a number that is higher by any number from 1 to 10, then Alan does the same,
and so on. Whoever calls 100 wins. How does Alan win?   [5]

9. A number is said to be palindromic if it reads the same backwards as forwards. For example,
7447 is palindromic. Show that all 4-digit palindromic numbers are divisible by eleven.   [5]

10. A polygon, not necessarily regular, is drawn so that its sides touch a given circle of radius r.
The perimeter of the polygon has length P. Find the area of the polygon in terms of r and P.   [5]

11. How many zeros are there at the end of the number which is the product of the first two
hundred positive integers? (i.e. the number called factorial 200 , written as 200!)   [5]

12. In a tetrahedron (triangular based pyramid) two opposite edges have the same length, a, and
are perpendicular to each other. Moreover they are each perpendicular to a line of length b
which joins their midpoints. Express the volume of the tetrahedron in terms of a and b.
(*Volume of a pyramid = $\frac{1}{3}$ x base area x height*)   [5]

13. Suppose that you walk slowly down a downward-moving escalator and take 50 steps of the
escalator to reach the bottom. Then by running five times as fast up the same escalator you
reach the top after taking 125 steps of the escalator. Assuming constant speeds for walking,
running and for the escalator, find how many steps the escalator shows when it is stationary.
                                                                                    [15]

14. Calculate the squares of: 67,   667,   6667 .

Guess the value of the square of the number consisting of one million sixes followed by a
seven. Now prove it.   [15]