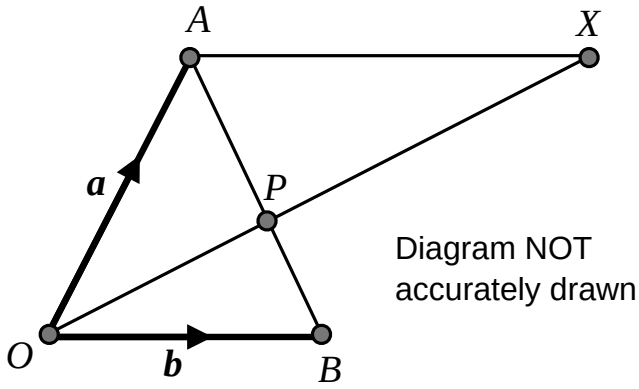
Chapter 9

GCSE and **A-Level Pure Mathematics**
Vectors I

9.1 Line Intersections

Some of the most difficult GCSE vector questions involve finding the position where two lines on a geometric figure intersect. They are hard, partly because they are being done without knowing the mathematical theory of vectors which is not studied until either the Further A-Level mathematics course or at University.

9.2 Example



OAB is a triangle with $\overrightarrow{OA} = \boldsymbol{a}$ and $\overrightarrow{OB} = \boldsymbol{b}$

P is a point on AB such that $AP : PB = 4 : 3$

Given that AX is parallel with OB and that OPX is a straight line, find $\overrightarrow{OX}$

Teaching Video : http://www.NumberWonder.co.uk/v9009/9a.mp4 (Part 1)
http://www.NumberWonder.co.uk/v9009/9b.mp4 (Part 2)

<= Part 1 Part 2 =>

Watch the videos and then answer the question in the space below.

☞

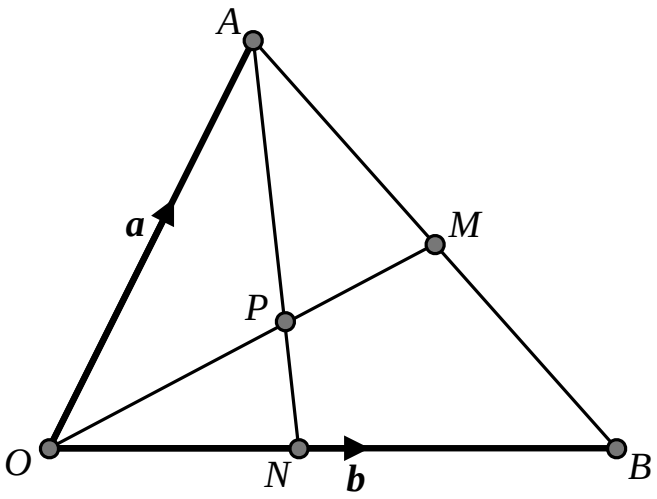
[5 marks]

9.3 Exercise

Any solution based entirely on graphical or numerical methods is not acceptable. Make the method used clear.

Marks available : 30

Question 1



OAB is a triangle in which $\overrightarrow{OA} = \boldsymbol{a}$ and $\overrightarrow{OB} = \boldsymbol{b}$

M is the midpoint of AB and OPM and APN are straight lines with $OP : PM = 4 : 3$

(i) Work out $\overrightarrow{OM}$ in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$

[1 mark]

(ii) Work out $\overrightarrow{OP}$ in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$

[1 mark]

(iii) Work out $\overrightarrow{AP}$ in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$

[1 mark]

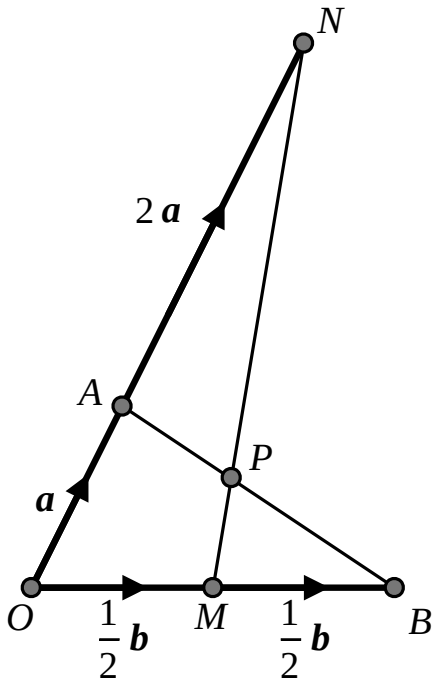
(iv) From the equation $\overrightarrow{AN} = \overrightarrow{AO} + \overrightarrow{ON}$ can be written that,

$s\overrightarrow{AP} = -\overrightarrow{OA} + t\overrightarrow{OB}$ for some constants s and t

Use this fact to work out the ratio $ON : NB$

[4 marks]

Question 2



In triangle OAB $\overrightarrow{OA} = \boldsymbol{a}$ and $\overrightarrow{OB} = \boldsymbol{b}$

OAN and MPN are straight lines

$OA : AN = 1 : 2$ and $OM : MB = 1 : 1$

(i) Work out $\overrightarrow{AB}$ in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$

[1 mark]

(ii) Work out $\overrightarrow{NM}$ in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$

[1 mark]

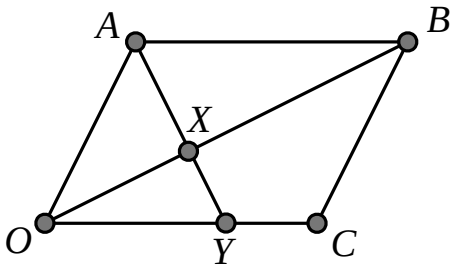
(iii) From the equation $\overrightarrow{AP} = \overrightarrow{AN} + \overrightarrow{NP}$ can be written that,

$s\overrightarrow{AB} = \overrightarrow{AN} + t\overrightarrow{NM}$ for some constants s and t

Use this fact to work out the ratio $AP : PB$

[3 marks]

Question 3



In parallelogram *OABC*, *Y* is the point on *OC* such that *OY* : *YC* = 2 : 1

$\overrightarrow{OA} = \boldsymbol{a}$ and $\overrightarrow{OC} = \boldsymbol{c}$

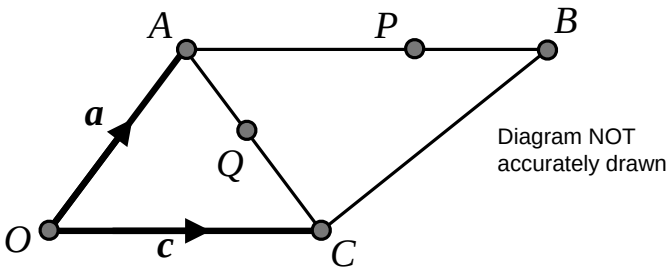
The diagonal *OB* intersects *AY* at *X*.

Calculate the ratio *AX* : *XY*

[6 marks]

Question 4

GCSE Examination Question from May 2019, Paper 1HR, Q24



$\overrightarrow{OA} = \boldsymbol{a}$ $\overrightarrow{OC} = \boldsymbol{c}$ $\overrightarrow{AB} = 2\boldsymbol{c}$

P is the point on AB such that $AP : PB = 3 : 1$

Q is the point on AC such that OQP is a straight line.

Use a vector method to find $AQ : QC$

Show your working clearly.

[5 marks]

Question 5

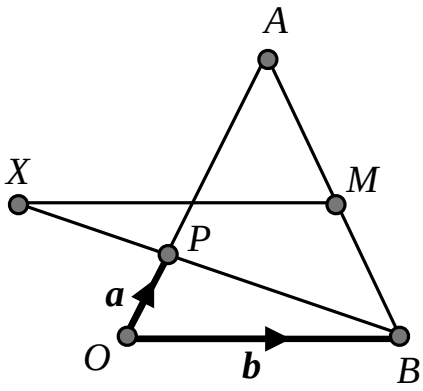


Diagram NOT accurately drawn

In triangle OAB, $\overrightarrow{OP} = \boldsymbol{a}$, $\overrightarrow{OA} = 3\boldsymbol{a}$ and $\overrightarrow{OB} = \boldsymbol{b}$

M is the midpoint of AB

(i) Express $\overrightarrow{BP}$ and $\overrightarrow{AB}$ in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$

[1 mark]

(ii) Express $\overrightarrow{MB}$ in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$

[1 mark]

(iii) If X lies on BP produced so that $\overrightarrow{BX} = k\overrightarrow{BP}$, express $\overrightarrow{MX}$ in terms of $\boldsymbol{a}$, $\boldsymbol{b}$ and k

[3 marks]

(iv) Find the value of k if MX is parallel to BO

[2 marks]