GCSE Mathematics

PROPORTIONALITY

On The Cover

The yacht pictured, a Vertue, is an example of a boat with a displacement hull.
It moves through the water by pushing the water to either side.
Notice that, because it's moving fairly fast, a large bow wave is being created.

A displacement hull has a speed beyond which it cannot pass through water.
As the hull speed approaches the theoretical maximum the energy from the sails makes the bow wave bigger rather than move the hull faster.

A displacement hull's maximum speed through the water, V_{max} is directly proportional to the square root of its waterline length, *LWL*.

The Vertue has a maximum theoretical speed of 6.3 knots.
Its *LWL* - Length Water Line - is 22 feet.

Lesson 1

Proportionality : GCSE

1.1 Direct Proportion

The topic of proportionality is about how mathematicians describe equations.

For GCSE the equations will typically involve two variables, say x and y.

In their mathematical description all of the following equations would invoke the words *directly proportional to*;

$$y = 6x \qquad y = 5\sqrt{x} \qquad y = 0.4x^2$$

and the numbers 6, 5 and 0.4 are each a *constant of the proportionality*.

In full we'd say;

$$y = 6x$$

y is directly proportional to x

The constant of the proportionality is 6

$$y = 5\sqrt{x}$$

y is directly proportional to the square root of x

The constant of the proportionality is 5

$$y = 0.4x^2$$

y is directly proportional to the square of x

The constant of the proportionality is 0.4

Example

Given that,

$$y = 9\sqrt{x}$$

(i) Tick which applies ☐ y is directly proportional to square root x
☐ y is inversely proportional to square root x

(ii) What is the constant of the proportionality ?

(iii) When $x = 49$, what is y ?

(iv) If the value of x was increased, would the value of y increase or decrease ?

1.2 Inverse Proportion

In their mathematical description all of the following equations would invoke the words *inversely proportional to*;

$$y = \frac{8}{x} \qquad y = \frac{9}{\sqrt{x}} \qquad y = \frac{3}{x^2}$$

and the numbers 8, 9 and 3 are each a *constant of the proportionality*.

In full we'd say;

$$y = \frac{8}{x} \quad or \quad y = 8 \times \frac{1}{x}$$

y is inversely proportional to x

The constant of the proportionality is 8

$$y = \frac{9}{\sqrt{x}} \quad or \quad y = 9 \times \frac{1}{\sqrt{x}}$$

y is inversely proportional to the square root of x

The constant of the proportionality is 9

$$y = \frac{3}{x^2} \quad or \quad y = 3 \times \frac{1}{x^2}$$

y is inversely proportional to the square of x

The constant of the proportionality is 3

Example
Given that,

$$y = \frac{20}{\sqrt{x}}$$

(i) Tick which applies ☐ y is directly proportional to square root x
☐ y is inversely proportional to square root x

(ii) What is the constant of the proportionality ?

(iii) When $x = 16$, what is y ?

(iv) If the value of x was increased, would the value of y increase or decrease ?

1.3 Exercise

Question 1

$$y = 7\sqrt{x}$$

(i) Tick which applies ☐ y is directly proportional to the square root of x
☐ y is inversely proportional to the square root of x

(ii) What is the constant of the proportionality ?

(iii) When $x = 9$, what is y ?

(iv) When $x = 25$, what is y ?

(v) If the value of x was increased, would the value of y increase or decrease ?

Question 2

$$y = \frac{36}{\sqrt{x}}$$

(i) Tick which applies ☐ y is directly proportional to the square root of x
☐ y is inversely proportional to the square root of x

(ii) What is the constant of the proportionality ?

(iii) When $x = 9$, what is y ?

(iv) When $x = 100$, what is y ?

(v) If the value of x was increased, would the value of y increase or decrease ?

Question 3

$$y = 0.5\ x^2$$

(i) Tick which applies ☐ y is directly proportional to the square of x
☐ y is inversely proportional to the square of x

(ii) What is the constant of the proportionality ?

(iii) When $x = 4$, what is y ?

(iv) When $x = 6$, what is y ?

(v) If the value of x was increased, would the value of y increase or decrease ?

Question 4

$$y = 100 \times \frac{1}{x^2}$$

(i) Tick which applies ☐ y is directly proportional to the square of x
☐ y is inversely proportional to the square of x

(ii) What is the constant of the proportionality ?

(iii) When $x = 2$, what is y ?

(iv) When $x = 5$, what is y ?

(iv) If the value of x was increased, would the value of y increase or decrease ?

Question 5

$$y = 8x$$

(i) Tick which applies ☐ y is directly proportional to x
☐ y is inversely proportional to x

(ii) What is the constant of the proportionality ?

(iii) When $x = 6$, what is y ?

(iv) When $x = 0.5$, what is y ?

(v) When $y = 32$, what is x ?

Question 6

$$y = \frac{6}{x}$$

(i) Tick which applies ☐ y is directly proportional to x
☐ y is inversely proportional to x

(ii) What is the constant of the proportionality ?

(iii) When $x = 2$, what is y ?

(iv) When $x = 12$, what is y ?

(v) When $y = 2$, what is x ?

Question 7

Write down the equation described by,

y is directly proportional to the square root of x

The constant of the proportionality is 0.4

Question 8

Write down the equation described by,

y is inversely proportional to the square of x

The constant of the proportionality is 12

Question 9

$$y = \frac{5m^2\sqrt{x}}{z}$$

(i) Tick which three apply

- ☐ *y* is directly proportional to square of *m*
- ☐ *y* is inversely proportional to square of *m*
- ☐ *y* is directly proportional to the square root of *x*
- ☐ *y* is inversely proportional to the square root of *x*
- ☐ *y* is directly proportional to *z*
- ☐ *y* is inversely proportional to *z*

(ii) What is the constant of the proportionality ?

(iii) When $m = 3$, $x = 100$, and $z = 50$ what is y ?

Question 10

$$y = \frac{2\sqrt{p}\,q}{v^2}$$

(i) Tick which three apply

- ☐ y is directly proportional to square root of p
- ☐ y is inversely proportional to square root of p
- ☐ y is directly proportional to q
- ☐ y is inversely proportional to q
- ☐ y is directly proportional to the square of v
- ☐ y is inversely proportional to the square of v

(ii) What is the constant of the proportionality ?

(iii) When $p = 25$, $q = 9$, and $v = 2$ what is y ?

Question 11

Write down the one equation described by,

y is directly proportional to the square of m

y is inversely proportional to w

y is directly proportional to the square root of s

The constant of the proportionality is 6

Question 12

Write down the equation described by,

y is directly proportional to the cube of x

The constant of the proportionality is 5

When x is 10, what is y ?

Question 13

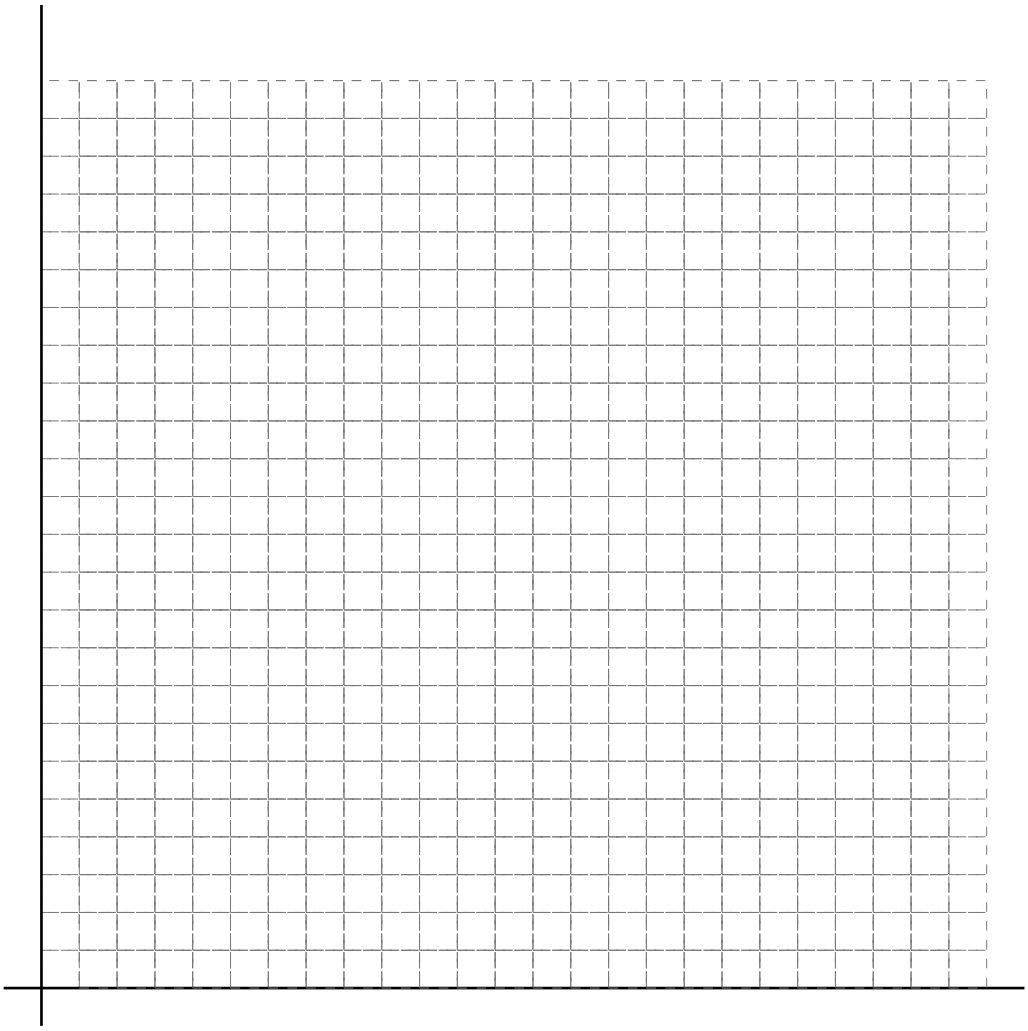
This question is about plotting a graph of direct proportion for the equation

$$y = 0.5\sqrt{x}$$

Complete the following table,

x	0	1	4	9	16	25
y						2.5

Plot the graph of the equation.



This direct proportion with a square root is sometimes described "diminishing returns". Explain this.