Lesson 5

GCSE : Algebra of Inequalities

5.1 Sandwich Inequalities

$$6\ cm\ <\ x\ <\ 11\ cm$$

The above inequality states that the unknown, x, has a value *between* 6 cm and 11 cm. We say that x is sandwiched between the values of 6 and 11 cm.

Sandwich inequalities are always written using $<$ or $\leqslant$
Not $>$ or $\geqslant$

5.2 Examples

$$\begin{array}{rl} -\ 10\ \leqslant\ 7x\ +\ 4\ <\ 39 & \\ -\ 14\ \leqslant\ 7x\ <\ 35 & -\ 4 \\ -\ 2\ \leqslant\ x\ <\ 5 & \div\ 7 \end{array}$$

$$\begin{array}{rl} 10\ \leqslant\ 43\ -\ 3x\ <\ 37 & \\ -\ 33\ \leqslant\ -3x\ <\ -6 & -\ 43 \\ -\ 11\ \leqslant\ -\ x\ <\ -\ 2 & \div\ 3 \\ 11\ \geqslant\ x\ >\ 2 & \times\ (-1) \\ 2\ <\ x\ \leqslant\ 11 & \textit{rewrite} \end{array}$$

5.3 Exercise

Do NOT use a calculator

Question 1

Solve the following inequalities

(i) $9\ <\ 2x + 3\ <\ 15$ **(ii)** $8\ <\ 3x\ -\ 4\ <\ 17$

(iii) $-8 < 5x + 2 \leqslant 47$

(iv) $-5 \leqslant 4x - 9 \leqslant 15$

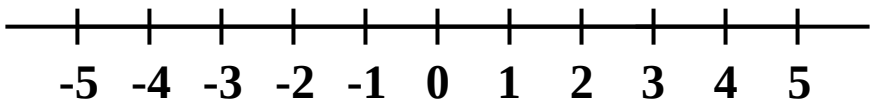
(v) $8 \leqslant \frac{x}{2} + 3 < 21$

(vi) $7 \leqslant \frac{x}{5} + 13 < 63$

Question 2

GCSE exam question, November 2007, 4H.

(a) On the number line, show the inequality $-2 < x \leqslant 3$



[2 marks]

(b) n is an integer.
Write down all the possible values of n which satisfy the inequality

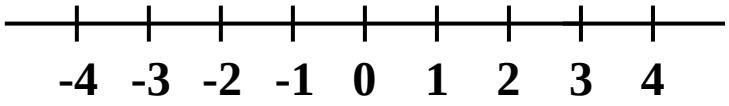
$$-1 \leqslant n < 4$$

[2 marks]

Question 3

(i) Solve the inequality $-10 \leqslant 3x - 7 < 2$

(ii) On the number line, represent the solution to part **(i)**.



Question 4

If x is an ***integer***, list all of the values of x for which it is true that;

$$-3.5 < x \leqslant 2$$

Question 5

GCSE exam question, November 2008, 4H.

Solve the inequality $5x - 6 > 2$

You must show sufficient working.

[2 marks]

Question 6

GCSE exam question from May 2008, 3H.

(i) Solve the inequality $4x + 5 \leqslant 21$

(ii) n is a positive integer.

Write down all the values of n which satisfy $4n + 5 \leqslant 21$

[4 marks]

Question 7

GCSE exam question from January 2012, 4H.

(i) Solve the inequalities

$$-6 < 4x < 8$$

(ii) n is an integer.

Write down all the values of n which satisfy

$$-6 < 4n \leqslant 8$$

[4 marks]

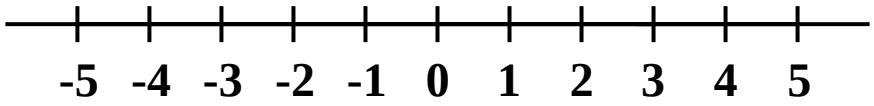
Question 8

GCSE exam question from January 2013, 3H

(i) Solve the inequalities

$$-2 < x + 2 \leqslant 5$$

(ii) On the number line, represent the solution to part (i).



[4 marks]

Question 9

GCSE exam question from November 2006, 3H.

Solve the inequality

$$9x - 2 < 5x + 4$$

[3 marks]

Question 10

Solve the following inequalities

(i) $31 \leqslant 3x + 25 < 37$

(ii) $-50 < 7x + 27 \leqslant 48$

(iii) $-8 < 8 - x \leqslant 28$

(iv) $-36 \leqslant 9x - 99 \leqslant 99$

(v) $9 \leqslant 3 - 2x < 23$

(vi) $9 \leqslant 13 - 4x < 65$

Question 11

(i) In this question x is an integer.
Also $x^2 < 25$.
List all possible values of x.

(ii) If y is not an restricted to being an integer, solve $y^2 < 25$.
Your answer should be in the form

$$a < y < b$$

for some values of a and b which you must find.

Question 12

GCSE exam question from May 2008, 4H.

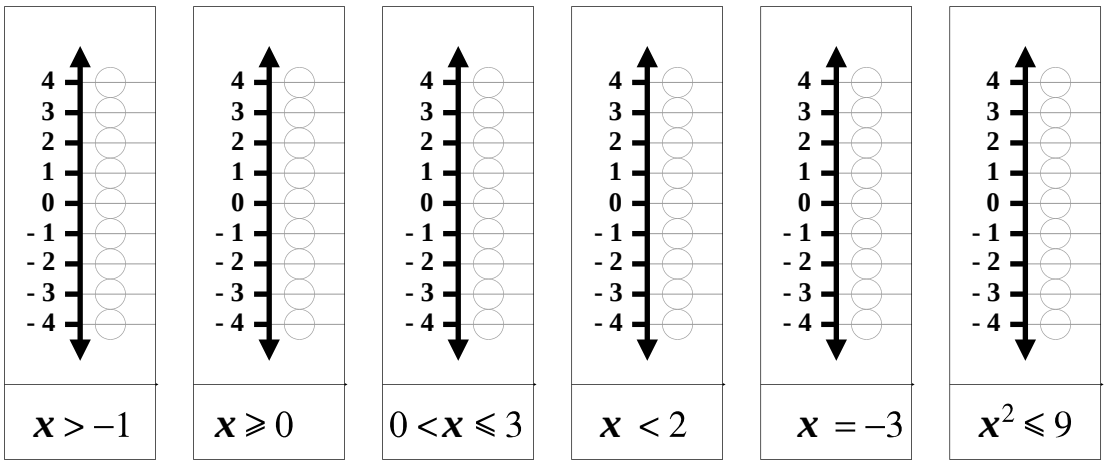
Solve the inequality

$$y^2 < 9$$

[2 marks]

Question 13

On these number ladders draw on the inequalities indicated



Question 14

GCSE exam question from June 2011, 4H.

(i) Solve the inequality

$$2x + 13 \geqslant 6$$

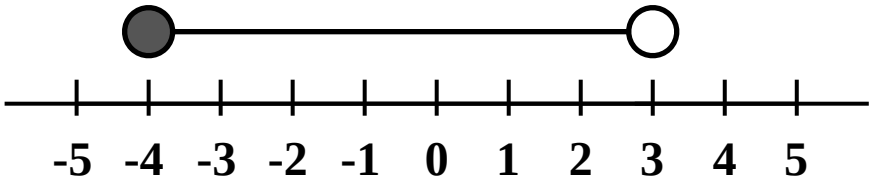
(ii) n is a **negative** integer.
Write down all the values of n which satisfy $2n + 13 \geqslant 6$

[4 marks]

Question 15

GCSE exam question from June 2010, 3H.

(a)



An inequality is shown on the number line.
Write down this inequality

(b) **(i)** Solve the inequality $2x + 9 > 1$

(ii) n is a **negative** integer.
Write down all the values of n which satisfy $2n + 9 > 1$

[6 marks]

Question 16

Write down all values of x that satisfy the following.

(i) $3x + 5 \leqslant 17$, where x is a positive, even integer.

(ii) $5x - 13 < 52$, where x is a positive, square number.

(iii) $7x + 17 < 141$, where x is a positive, prime number.