

Appendix : Completing the Square

How to solve quadratics by Martin Hansen
(Published in M500, The Open University Mathematics Magazine)

To solve the quadratic equation:

$$ax^2 + bx + c = 0$$

Multiply through by $4a$:

$$\left[4a^2x^2 + 4abx \right] + 4ac = 0$$

Complete the square within the square brackets:

$$\left[(2ax + b)^2 - b^2 \right] + 4ac = 0$$

$$(2ax + b)^2 = b^2 - 4ac$$

Square-root both sides:

$$2ax + b = \pm \sqrt{b^2 - 4ac}$$

And the well-known formula is there for the taking:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$