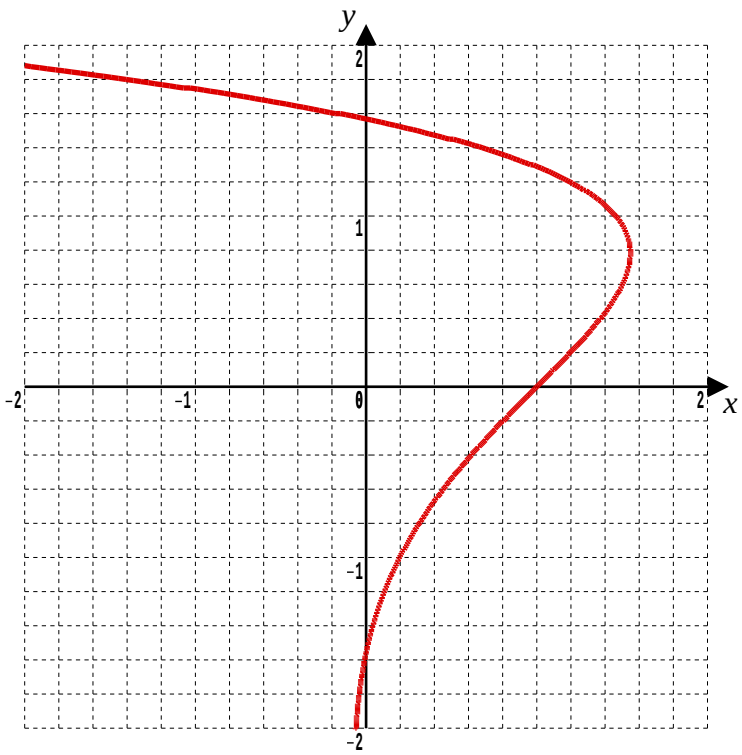
Lesson 10

A-Level Pure Mathematics : Year 2

Differentiation III

10.1 Differentiating $x = f(y)$

The graph is of the curve with equation $x = e^y \cos y$ with y in radians.



Video : http://www.NumberWonder.co.uk/v9028/10a.mp4

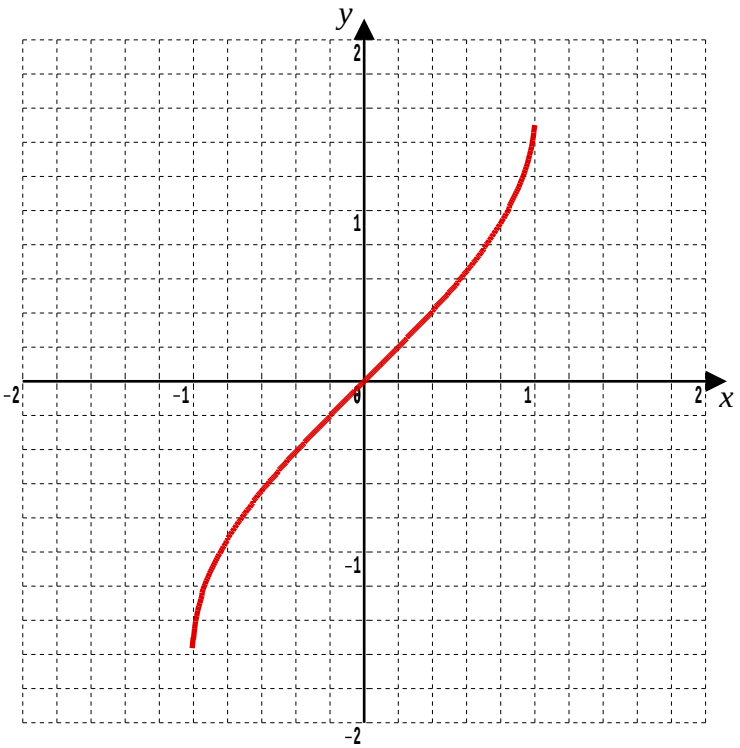
(i) Obtain an expression for $\frac{dy}{dx}$ in terms of y

(ii) What is the equation of the normal to the curve when $y = 0$?

(iii) Add your part (ii) normal onto the graph above

[2, 2, 2 marks]

10.2 Differentiating *arcsin x*



The graph of $y = arcsin\, x$
It is the inverse of a one-to-one piece of the *sin x* function

$$y = arcsin\, x \Rightarrow \frac{dy}{dx} = \frac{1}{\sqrt{1 - x^2}}$$

Proof
Teaching Video : http://www.NumberWonder.co.uk/v9028/10b.mp4

Watch the video and
then write out the
proof here

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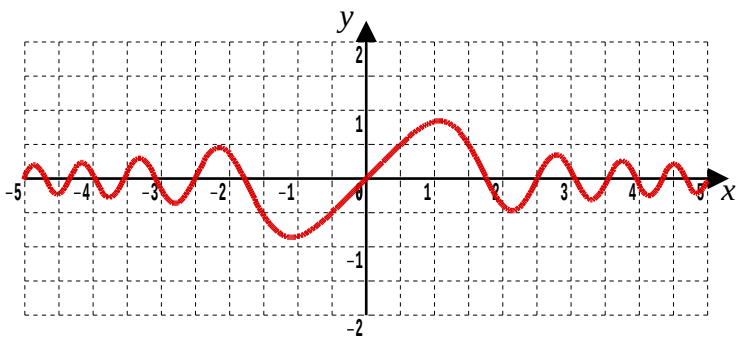
[6 marks]

10.3 Exercise

Marks Available : 40

Question 1

The graph is of the curve with equation $y = \dfrac{sin(x^2)}{x}$ with x in radians.



(i) Obtain an expression for $\dfrac{dy}{dx}$ in terms of x

[3 marks]

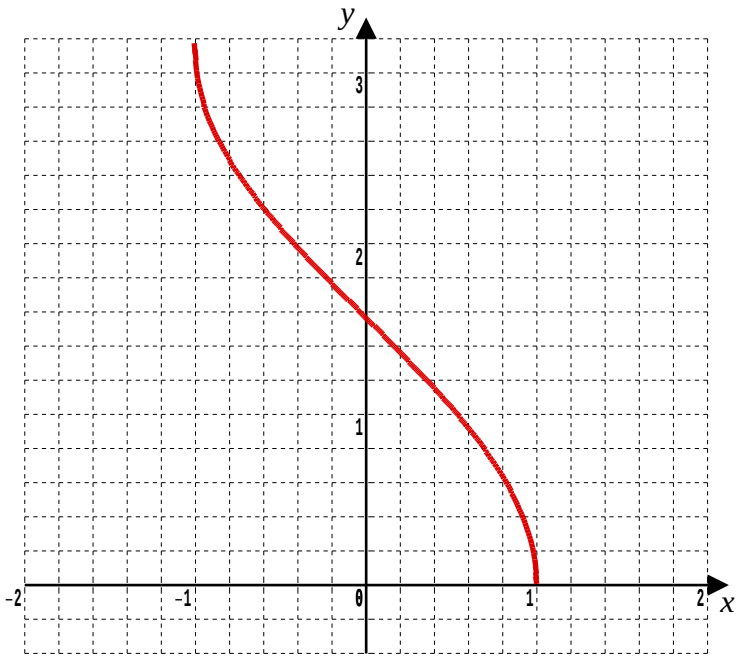
(ii) What is the ***exact*** equation of the tangent to the curve when $x = \sqrt{\pi}$

[5 marks]

(iii) Add your part (ii) tangent onto the graph above

[1 mark]

Question 2



The graph of $y = \arccos x$

It is the inverse of a one-to-one piece of the $\cos x$ function

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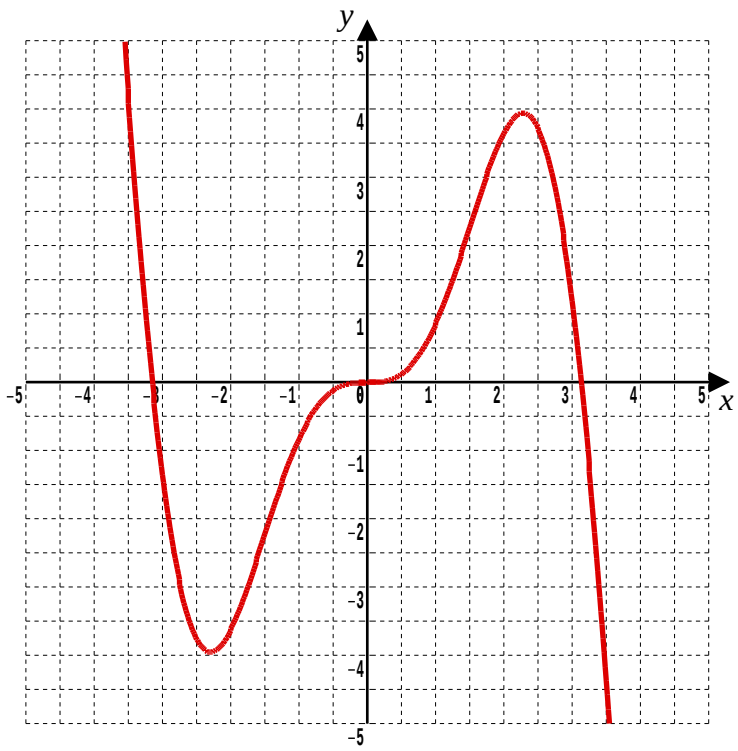
$$y = \arccos x \Rightarrow \frac{dy}{dx} = -\frac{1}{\sqrt{1 - x^2}}$$

Assuming standard results for $\sin x$ and $\cos x$ prove the above result.

[7 marks]

Question 3

The graph is of a piece of the curve with equation $y = x^2 \sin x$



(i) Obtain an expression for $\frac{dy}{dx}$ in terms of x

[3 marks]

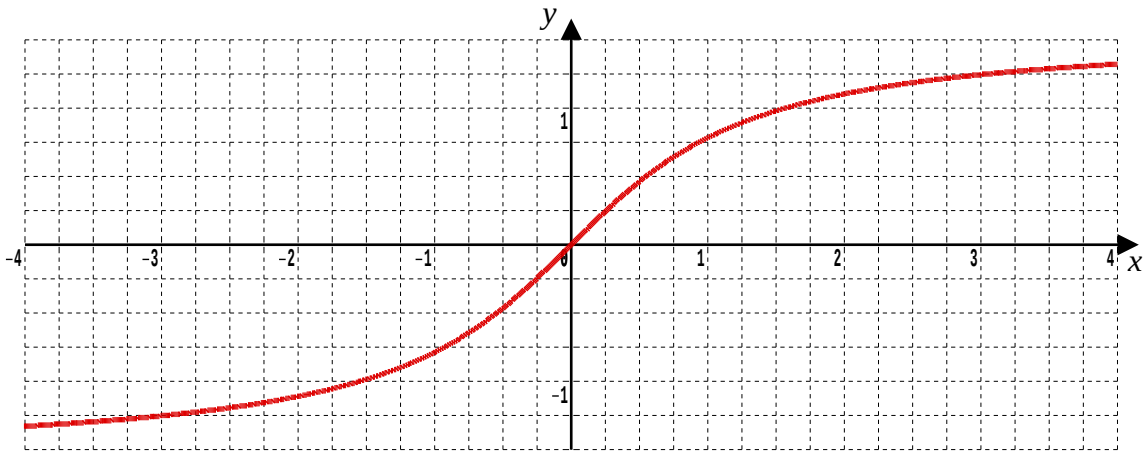
(ii) What is the ***exact*** equation of the normal to the curve when $x = \pi$

[6 marks]

(iii) Add your part (ii) normal onto the graph above

[1 mark]

Question 4



The graph of $y = \arctan x$

It is the inverse of a one-to-one piece of the tan x function

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$$y = \arctan x \Rightarrow \frac{dy}{dx} = \frac{1}{1 + x^2}$$

(i) Show that if $y = \tan x$ then $\frac{dy}{dx} = \sec^2 x$ by using the derivatives of $\sin x$ and $\cos x$ and the quotient rule.

[2 marks]

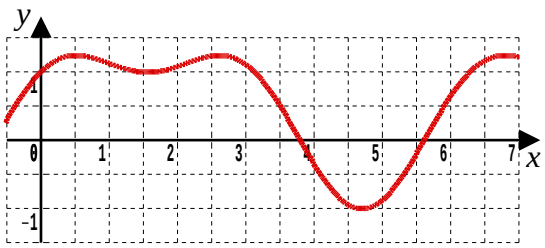
(ii) Hence prove that if $y = \arctan x$ then $\frac{dy}{dx} = \frac{1}{1 + x^2}$

[6 marks]

Question 5

A curve has equation, $y = \cos^2 x + \sin x \qquad 0 < x < 2\pi$

Find the coordinates of its stationary points.



[6 marks]