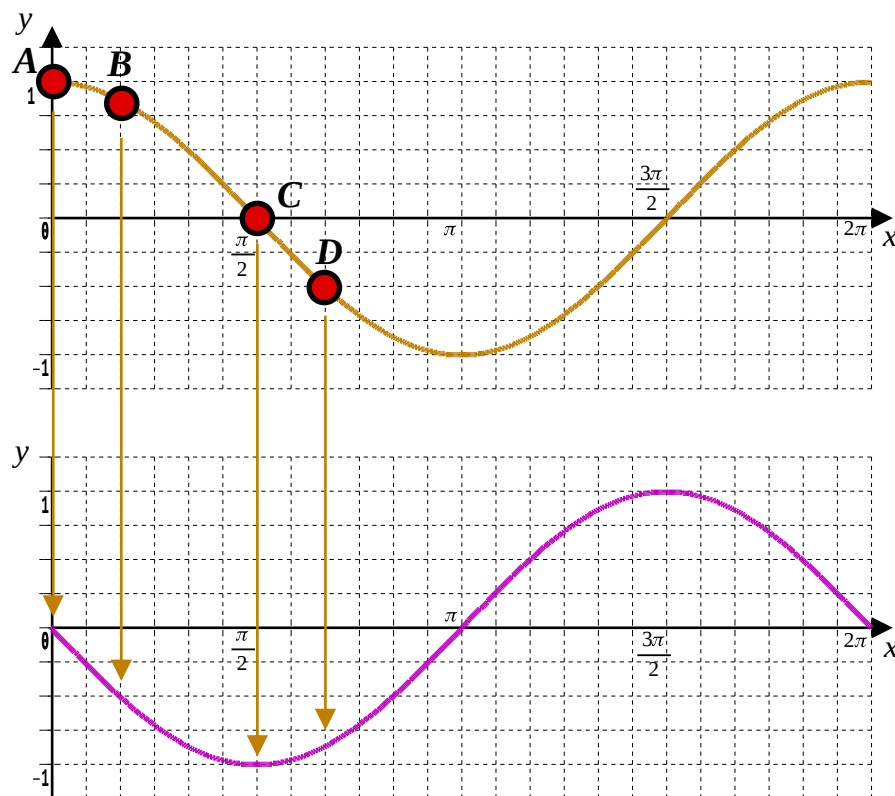


9.1 The Cosine Function

The upper graph shows the curve $y = \cos x$ with x measured in radians. Of interest is the gradient of this graph.



Consider what the gradient of the upper graph is doing at the four points labelled.

A : At this maximum turning point the gradient is zero.

B : The gradient between A and B is negative and increasing in magnitude.

C : The curve seems to be sloping like the line $y = -x$, gradient -1

D : The gradient between C and D is negative and decreasing in magnitude.

Continuing in this manner, and plotting the gradients as a separate graph, the lower graph is obtained. It looks very much like the graph of $-\sin x$

This argument, although not a proof, is an intuitive reasoning of the important result that, provided radians are used,

Derivative of Cosine

$$\text{If } y = \cos x \text{ then } \frac{dy}{dx} = -\sin x \quad \text{where } x \text{ is in radians}$$

9.2 Differentiating Cosine Functions

The Product Rule and The Quotient Rule can be applied to situations where the cosine function is involved. So too can The Chain Rule, as follows,

The Chain Rule for $y = \cos(f(x))$

$$\text{If } y = \cos(f(x)) \text{ then } \frac{dy}{dx} = -\sin(f(x)) \times f'(x)$$

9.3 Example

Prove that a consequence of the derivative of $\sin x$ being $\cos x$ is that the derivative of $\cos x$ is $(-\sin x)$

Teaching Video : <http://www.NumberWonder.co.uk/v9028/9.mp4>



Watch the video and
then write out the
proof here



[6 marks]

9.4 Exercise

Marks Available : 40

Question 1

Differentiate each of the following with respect to x ,

(i) $y = \cos(3x^5 + 2e^{3x})$ (ii) $y = 5x^7 + \cos^4(3x)$

[2, 2 marks]

(iii) $y = \cos\left(\frac{5}{x} + \ln x\right), \quad x > 0$ (iv) $y = \sin(1 - \cos x)$

[2, 2 marks]

Question 2

By writing $y = \sec x$ as $y = (\cos x)^{-1}$ and using The Chain Rule, show that,

Derivative of $\sec x$

$$\text{If } y = \sec x \text{ then } \frac{dy}{dx} = \sec x \tan x \quad \text{where } x \text{ is in radians}$$

[3 marks]

Question 3

By writing $y = \tan x$ as $y = \frac{\sin x}{\cos x}$ and using The Quotient Rule, show that,

Derivative of $\tan x$

If $y = \tan x$ then $\frac{dy}{dx} = \sec^2 x$ where x is in radians

[3 marks]

Question 4

By writing $y = \cot x$ as $y = \frac{\cos x}{\sin x}$ and using The Quotient Rule, show that,

Derivative of $\cot x$

If $y = \cot x$ then $\frac{dy}{dx} = -\csc^2 x$ where x is in radians

[3 marks]

Question 5**Table of Derivatives of Trigonometric Functions**

$f(x)$	$f'(x)$	Told in Exam ?
$\sin x$	$\cos x$	No
$\cos x$	$-\sin x$	No
$\csc x$	$-\csc x \cot x$	Yes
$\sec x$	$\sec x \tan x$	Yes
$\tan x$	$\sec^2 x$	Yes
$\cot x$	$-\csc^2 x$	Yes

Use the above table of standard derivatives to differentiate each of the following with respect to x ,

(i) $y = \tan(5x)$

(ii) $y = \sec(3x^4 + x^2)$

[2, 2 marks]

(iii) $y = \cot(5 - \ln x)$

(iv) $y = e^{\tan 4x}$

[2, 2 marks]

(v) $y = \ln(\sec x + \tan x)$

Simplify your answer as much as possible

[2 marks]

Question 6

From the small angle approximations it is known that,

$$\bullet \quad \sin \theta \approx \theta \quad \bullet$$

$$\bullet \quad \cos \theta \approx 1 - \frac{\theta^2}{2} \quad \bullet$$

$$\bullet \quad \tan \theta \approx \theta \quad \bullet$$

This allows a couple of useful deductions to be made, firstly that,

$$\lim_{h \rightarrow 0} \frac{\sin h}{h} = \lim_{h \rightarrow 0} \frac{h}{h} = 1$$

and secondly, that,

$$\lim_{h \rightarrow 0} \frac{\cos h - 1}{h} = \lim_{h \rightarrow 0} \frac{1 - \frac{1}{2}h^2 - 1}{h} = \lim_{h \rightarrow 0} \left(-\frac{1}{2}h \right) \rightarrow 0$$

(i) Given that $f(x) = \cos x$, show from first principles that,

$$f'(x) = \lim_{h \rightarrow 0} \left(\left(\frac{\cos h - 1}{h} \right) \cos x - \frac{\sin h}{h} \sin x \right)$$

[4 marks]

(ii) Hence prove that $f'(x) = -\sin x$

[2 marks]

Question 7

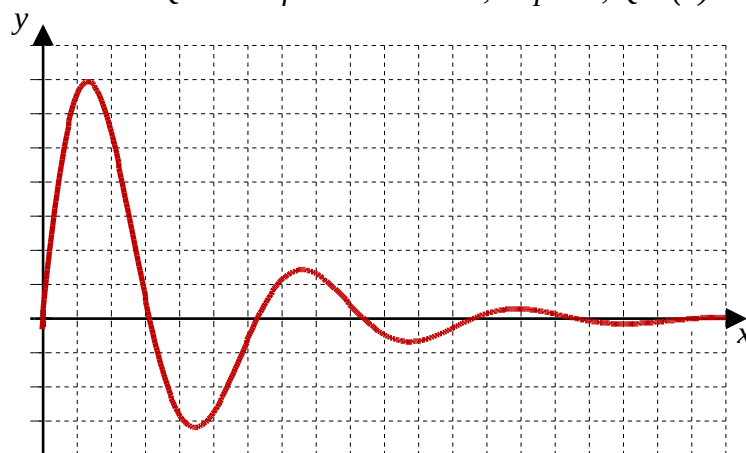
Given that $f(x) = \csc x$ find the exact value of $f'\left(\frac{\pi}{6}\right)$

Simplify your answer.

[3 marks]

Question 8

A-Level Examination Question from June 2019, Paper 1, Q12(a)



$$f(x) = 10 e^{-0.25x} \sin x \quad x \geq 0$$

Show that the x coordinates of the turning points of the curve with equation $y = f(x)$ satisfy the equation $\tan x = 4$

[4 marks]