Lesson 8

A-Level Pure Mathematics : Year 2
Trigonometric Identities

8.1 Waveforms Expressed as *R sin* ($\theta \pm \alpha$) or *R cos* ($\theta \pm \alpha$)

In lesson 7 we looked at rewriting expressions of the form

$$a \sin\theta + b \cos\theta$$

as

$$R \sin(\theta + \alpha)$$

This is one of four possible variations on the same theme.

Here are all four rewrites, given that a and b are positive valued;

- $a \sin\theta \pm b \cos\theta \equiv R \sin(\theta \pm \alpha)$
- $a \cos\theta \pm b \sin\theta \equiv R \cos(\theta \mp \alpha)$

with $R > 0$ and $0 < \alpha < 90°$

where $R \cos\alpha = a$ and $R \sin\alpha = b$ and $R = \sqrt{a^2 + b^2}$

The recommended method of tackling problems involving such rewrites is to, as we did in lesson 7, use the addition formula to expand whichever of $\sin(\theta \pm \alpha)$ or $\cos(\theta \mp \alpha)$ is desired, then equate coefficients of $\sin\theta$ and $\cos\theta$.

8.2 Example

Express $4 \cos\theta + 3 \sin\theta$ in the form $R \cos(\theta - \alpha)$ for $R > 0$ and $0 < \alpha < 90°$

Teaching Video : http://www.NumberWonder.co.uk/v9040/8.mp4

After watching the video
write out a solution to
the example

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[4 marks]

Note : Rearranging the expression as $3 \sin\theta + 4 \cos\theta$ gives rise to a rewrite of $5 \sin(\theta + 53.1°)$. The two answers are equivalent because $\cos\varphi = \sin(\varphi + 90°)$

8.3 Exercise

Any solution based entirely on graphical
or numerical methods is not acceptable
Marks Available : 40

Question 1

(i) Express $15\,cos\,\theta + 36\,sin\,\theta$ in the form $R\,cos\,(\,\theta - \alpha\,)$

where $R > 0$ and $0 < \alpha < 90°$

[3 marks]

(ii) Hence solve, for $0 < \theta < 360°$, the equation $15\,cos\,\theta + 36\,sin\,\theta = 13$

[3 marks]

Question 2

C3 Examination Question from January 2006, Q6

$$f(x) = 12\cos x - 4\sin x$$

Given that $f(x) = R\cos(x + \alpha)$, where $R \geqslant 0$ and $0 \leqslant \alpha \leqslant 90°$

(a) find the value of R and the value of α

[4 marks]

(b) Hence solve the equation $12\cos x - 4\sin x = 7$

for $0 \leqslant x \leqslant 360°$, giving your answer to one decimal place.

[5 marks]

(c) **(i)** Write down the minimum value of $12\cos x - 4\sin x$

[1 mark]

(ii) Find, to 2 decimal places, the smallest positive value of x for which this minimum value occurs.

[2 marks]

Question 3

C3 Examination Question from January 2009, Q8

(a) Express $3\cos\theta + 4\sin\theta$ in the form $R\cos(\theta - \alpha)$, where R and α are constants, $R > 0$ and $0 < \alpha < 90°$

[4 marks]

(b) Hence find the maximum value of $3\cos\theta + 4\sin\theta$ and the smallest positive value of θ for which this maximum occurs.

[3 marks]

The temperature, $f(t)$, of a warehouse is modelled using the equation

$$f(t) = 10 + 3\cos(15t) + 4\sin(15t)$$

where t is the time in hours from midday and $0 \leqslant t < 24$

(c) Calculate the minimum temperature of the warehouse as given by this model.

[2 marks]

(d) Find the value of t when this minimum temperature occurs.

[3 marks]

Question 4

(a) Express

$$5\,sin^2\,\theta\;-\;3\,cos^2\,\theta\;+\;6\,sin\,\theta\,cos\,\theta$$

in the form

$$a\,sin\,2\theta\;+\;b\,cos\,2\theta\;+\;c$$

where a, b and c are constants to be found.

[3 marks]

(b) Hence find the maximum and minimum values of

$$5\,sin^2\,\theta\;-\;3\,cos^2\,\theta\;+\;6\,sin\,\theta\,cos\,\theta$$

[3 marks]

(c) Solve $5\,sin^2\,\theta\ -\ 3\,cos^2\,\theta\ +\ 6\,sin\,\theta\,cos\,\theta\ =\ -1$

for $0\ \leqslant\ \theta\ \leqslant\ 180$, rounding your answers to 1 decimal place.

[4 marks]