

4.1 The Modulus Function (Part 2)

The fact that the modulus function can take any part of an equation's graph that is below the x -axis, and reflects it in the x -axis was studied previously. Here is the rule that was used;

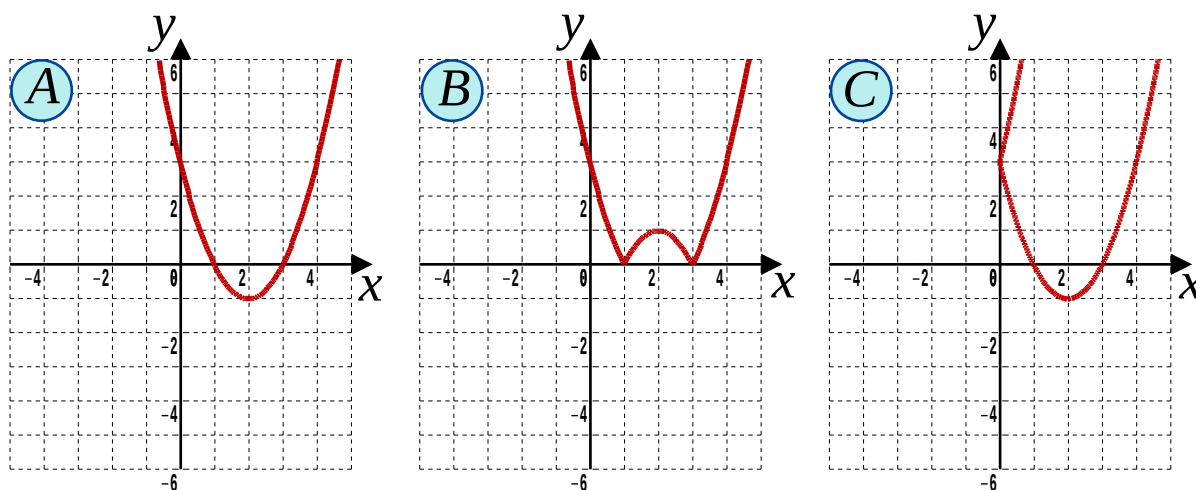
Modulus Sketching Rule 1

To sketch $y = |f(x)|$

Bounce negative x

- ◇ Sketch $y = f(x)$ using a dashed line for points below the x -axis.
- ◇ Reflect any part of the curve below the x -axis in the x -axis.

For example, graph A shows the parabola with quadratic equation $y = x^2 - 4x + 3$ and graph B shows the effect of modulating graph A: That is, $y = |x^2 - 4x + 3|$

**4.1.1 Example**

Find the equation that will yield graph C, where the “bounce” is against the y -axis.

4.2 By Hand

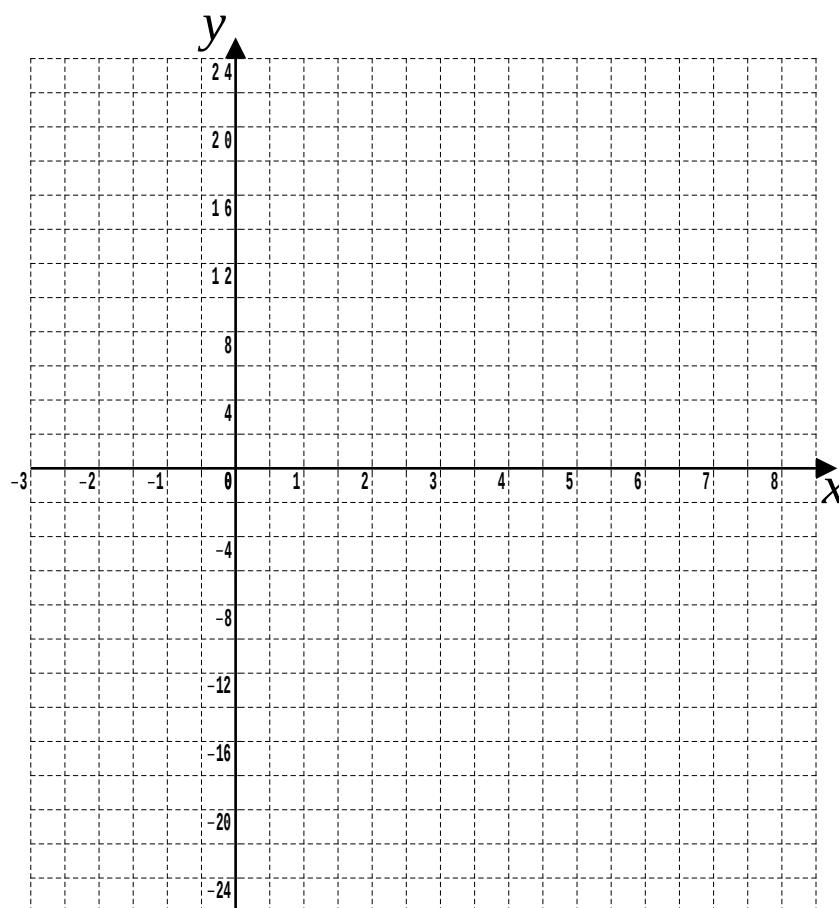
Faced with an unfamiliar modulus question it may be best to go back to basics by drawing up a table and plotting points to get a feel of how it is behaving. With such plotting, keeping one's wits sharp is essential !

4.2.1 Example

Use the table below to assist in plotting the following equation;

$$|y + 1| = |(x - 2)^2|$$

x	-3	-2	-1	0	1	2	3	4	5	6	7
y											



[6 marks]

4.3 A Subtle Variation

Typically, an examination question will ask for two separate graphs of

$$\begin{array}{l} \diamond \quad y = |f(x)| \\ \text{and} \quad \diamond \quad y = f(|x|) \end{array}$$

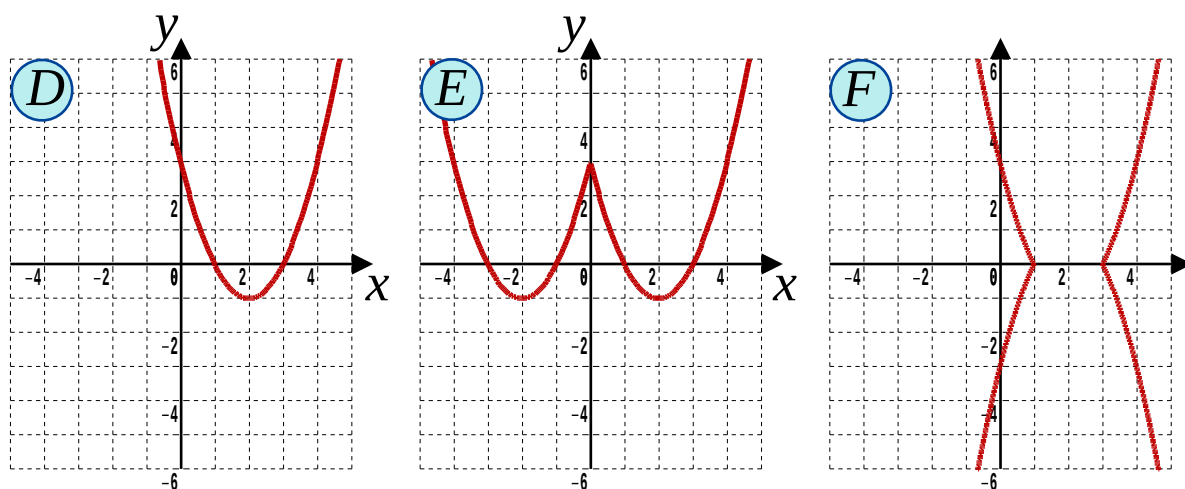
Modulus Sketching Rule 2

To sketch $y = f(|x|)$

Overwrite negative x

- $\diamond$ Sketch the curve of $y = f(x)$ for $x \geq 0$
- $\diamond$ Reflect the parts of the curve to the right of the y -axis in the y -axis.

Graph D shows the parabola with quadratic equation $y = x^2 - 4x + 3$ and graph E shows the effect of modulsing each individual x . That is, $y = |x^2| - 4|x| + 3$. Notice that the information to the left of the y -axis of the original graph has been overwritten, whilst information to the right has been “duplicated” in a reflected form.



4.3.1 Example

Find the equation that will yield graph F , where the “mirror” is in the x -axis.

[2 marks]

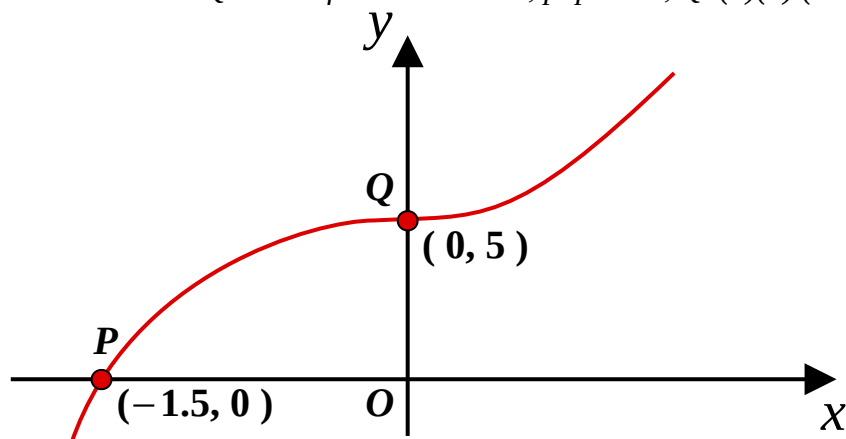
4.4 Exercise

*Any solution based entirely on graphical
or numerical methods is not acceptable*

Marks Available: 100

Question 1

A-Level Examination Question from June 2012, paper C3, Q4(a)(b) (Edexcel)



The curve $y = f(x)$ passes through the points $P(-1.5, 0)$ and $Q(0, 5)$ as shown.

On separate diagrams, sketch the curve with equation,

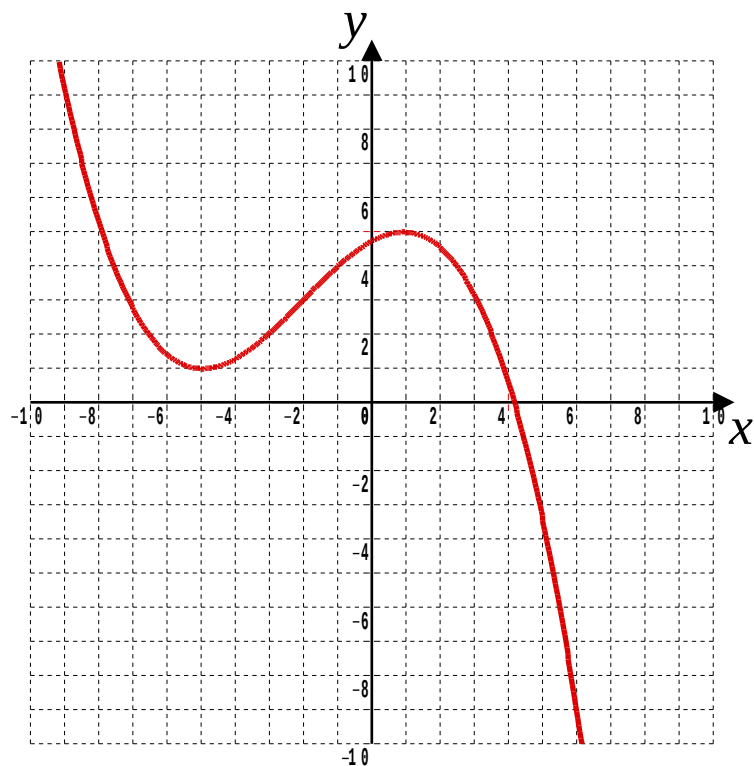
(a) $y = |f(x)|$

(b) $y = f(|x|)$

Indicate clearly on each sketch the coordinates of the points at which the curve crosses or meets the axes.

[2, 2 marks]

Question 2



The diagram shows the graph of $y = f(x)$

The point $(1, 5)$ is the maximum turning point of the graph.

The point $(-5, 1)$ is the minimum turning point of the graph.

Sketch, on separate diagrams, the graphs of

(i) $y = |f(x)|$

(ii) $y = f(|x|)$

Show on each graph the coordinates of any turning points.

[2, 3 marks]

Question 3

$$h(x) = 2(x - 3)^2 - 8, \quad x \in \mathbb{R}$$

- (a)** For each of the following, draw a sketch.
Label any turning points and axes intercepts.

(i) $y = h(x)$ **(ii)** $y = |h(x)|$ **(iii)** $y = h(|x|)$

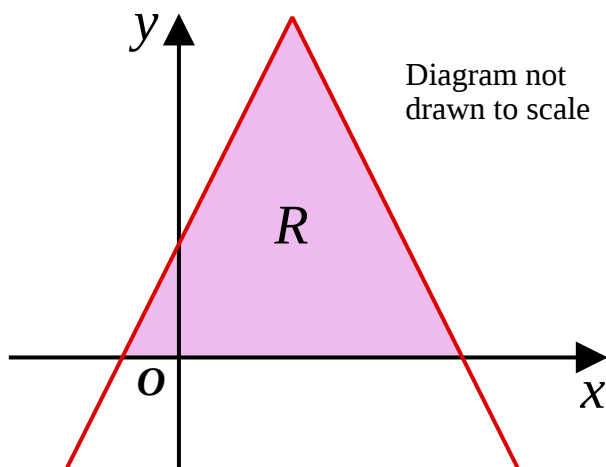
[3, 3, 3 marks]

- (b)** For what values of k will the equation $h(|x|) = k$ have four solutions ?

[2 marks]

Question 4

A-Level Mock Paper 2 from 2019, Q5 (Edexcel)



The graph is part of the equation $y = f(x)$ where $f(x) = 7 - |3x - 5|$, $x \in \mathbb{R}$

The finite region R , shown shaded, is bounded by the graph $y = f(x)$ and the x -axis.

(a) Find the area of R , giving your answer in its simplest form.

[4 marks]

The equation $7 - |3x - 5| = k$ where k is a constant has two distinct real solutions.

(b) Write down the range of possible values for k

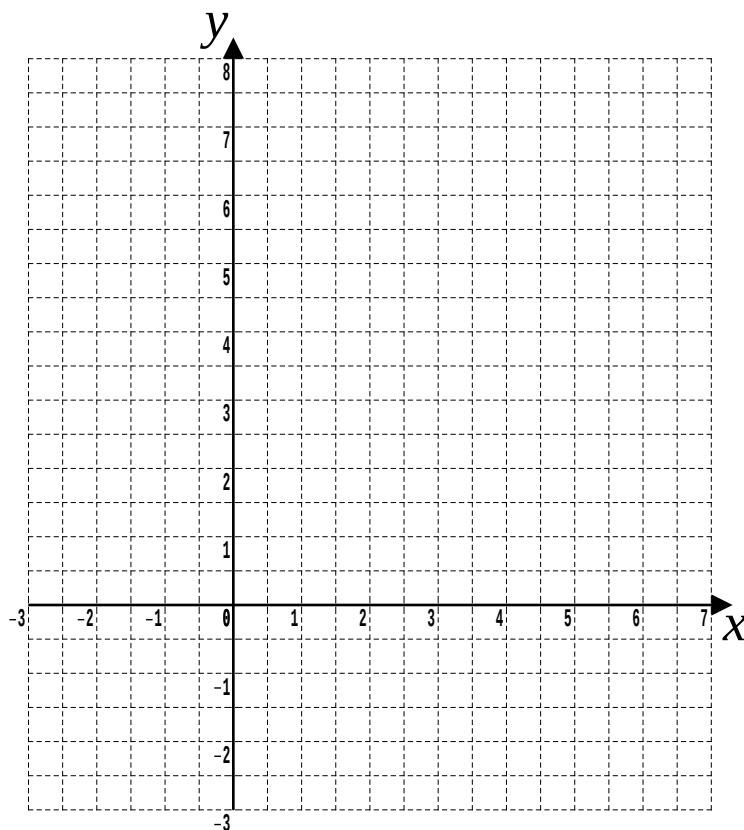
[1 mark]

Question 5

- (i) Use the table below to assist in plotting the following equation;

$$|y - 3| = |x - 2|$$

x	-3	-2	-1	0	1	2	3	4	5	6	7
y											



[5 marks]

- (ii) Given that the following equation has exactly one real solution, find k ;

$$|y - 3| - |x - 2| = k$$

[1 mark]

- (iii) Given that the following equation has exactly one real solution, find c ;

$$|y - 3| - |x - 2| = \frac{1}{4}x + c$$

[1 mark]

Question 6

$$f(x) = |x|^2 - 4|x| + 1 \quad x \in \mathbb{R}$$

(a) Given that x is a real number, which of the following are true ?

(i) $|x|^2 = x^2$ **(ii)** $|x^2| = x^2$ **(iii)** $|x^2| = |x|^2$

[3 marks]

(b) Sketch the curve, $y = f(x)$, marking on the coordinates of any local minima or maxima.

[3 marks]

(c) Using algebra, solve the equation, $f(x) = -2$

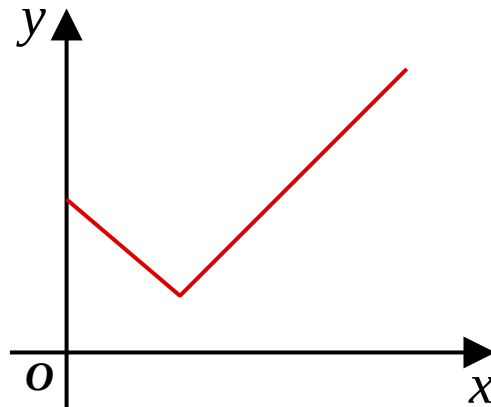
[3 marks]

(d) If $f(x) = k$, where k is a constant, has exactly two solutions, state the possible values of k

[3 marks]

Question 7

A-Level Sample Assessment Materials from 2017, Paper 2, Q11 (Edexcel)



The graph is a sketch of $y = f(x)$ where $f(x) = 2|3 - x| + 5$, $x \geq 0$

(a) State the range of f

[1 mark]

(b) Solve the equation, $f(x) = \frac{1}{2}x + 30$

[3 marks]

Given that the equation $f(x) = k$, where k is a constant, has two distinct roots,

(c) state the set of possible values for k

[2 marks]

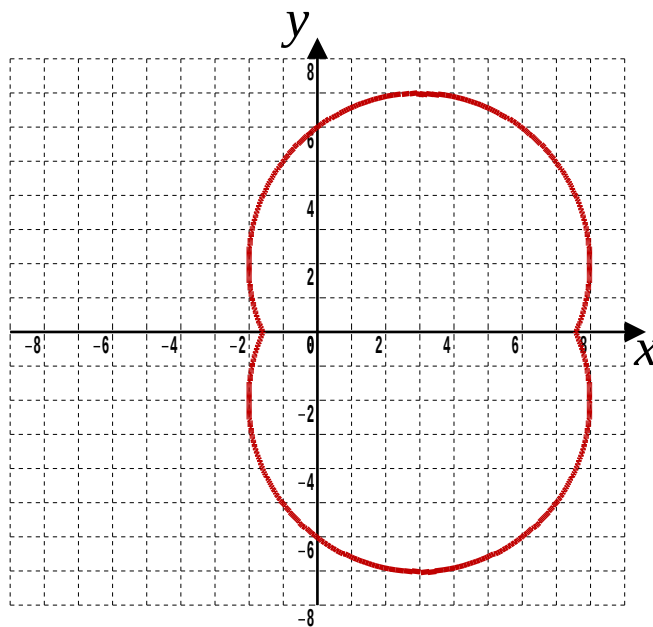
Question 8

A circle centre $(3, 2)$ and radius 5 has equation $(x - 3)^2 + (y - 2)^2 = 25$

This equation has been manipulated in various ways and the modulus function applied to obtain the following graphs on a computer capable of processing the $\pm$ sign in an input equation.

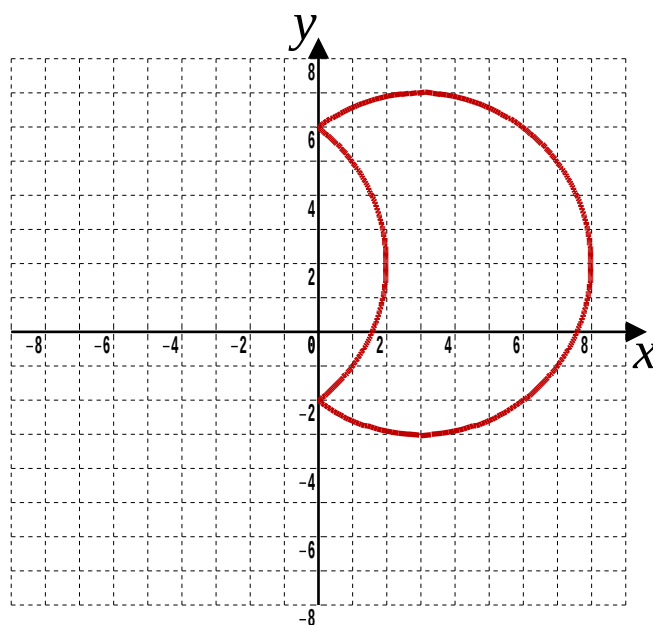
Underneath each graph write the equation used to produce the graph.

(i)



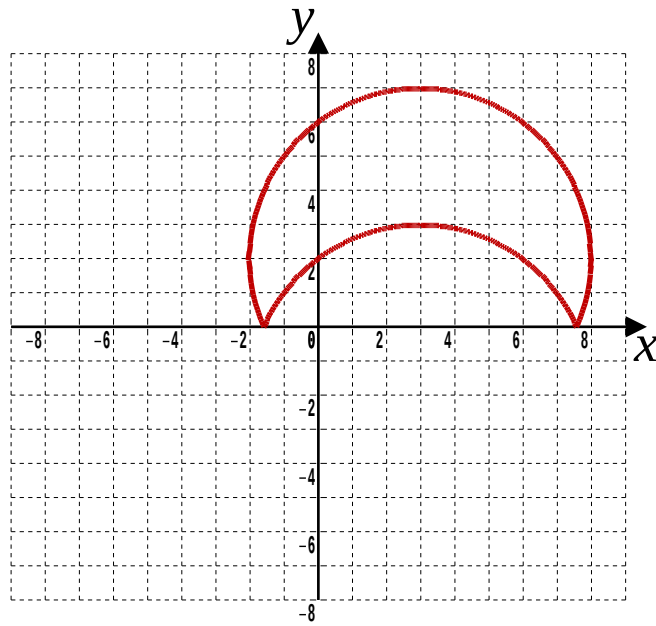
[3 marks]

(ii)



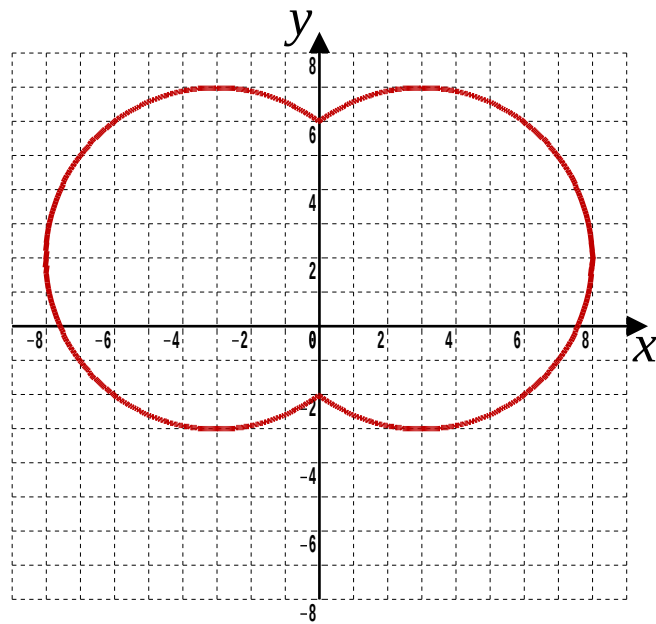
[3 marks]

(iii)



[3 marks]

(iv)



[3 marks]

If you have a graphics calculator you may like to see if you can get it to plot each of these curves. Where a $\pm$ is involved you may have to plot the equation with the plus separately to the equation with a minus.

Question 9

$$f(x) = 3x^2 - 12x + 7, \quad x \in \mathbb{R}$$

- (a) Write $f(x)$ in the form $a(x + b)^2 + c$, where a , b and c are integers.

[3 marks]

- (b) Sketch the curve with equation $y = f(x)$ showing any points of intersection with the coordinate axes and the coordinates of any turning point.

[3 marks]

- (c) (i) Describe fully the transformation that maps the curve with equation $y = f(x)$ onto the curve with equation $y = g(x)$ where,

$$g(x) = 3(x - 1)^2 - 12x + 9, \quad x \in \mathbb{R}$$

[2 marks]

- (ii) Find the range of the function

$$h(x) = \frac{20}{3x^2 - 12x + 7} \quad x \in \mathbb{R}$$

[2 marks]

Question 10

A-Level Examination Question from January 2014, Paper C3, Q6 (Edexcel)

Given that a and b are constants and that $0 < a < b$,

(a) on separate diagrams, sketch the graph with equation

(i) $y = |2x + a|$ **(ii)** $y = |2x + a| - b$

Show on each sketch the coordinates of each point at which the graph crosses or meets the axes.

[6 marks]

(b) Solve, for x , the equation

$$|2x + a| - b = \frac{1}{3}x$$

giving any answers in terms of a and b

[4 marks]

Question 11

The functions f and g are defined as,

$$f(x) = 4a^2 - x^2, \quad x \in \mathbb{R}$$

$$g(x) = |4x - a|, \quad x \in \mathbb{R}$$

where a is a constant, such that $a \geq 1$

- (i) Sketch the graph of $f(x)$ and the graph of $g(x)$ on the same diagram.

The sketch must include the coordinates of any points where each of the graphs meets the coordinate axes.

[6 marks]

- (ii) Find, in exact form where appropriate, the solutions of the equation,

$$4 - x^2 = |4x - 1|$$

[4 marks]

Question 12

Consider the function

$$f(x) = \frac{\pi}{180}x - 5 + \sin x, \quad -300^\circ \leq x \leq 600$$

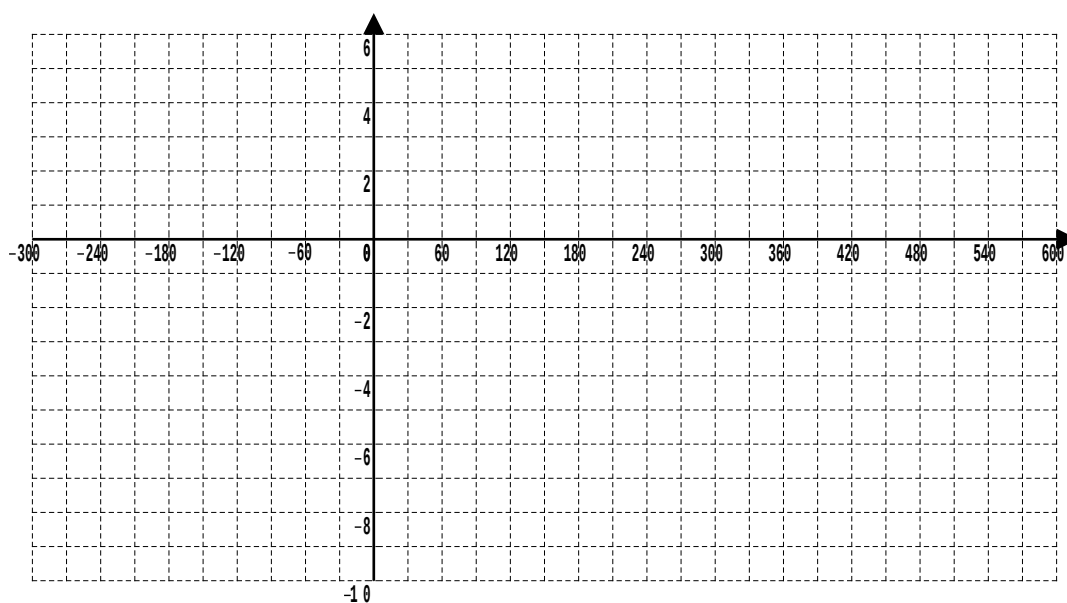
(a) Complete the following table,

x	-300	-240	-180	-120	-60	0	60	120
y								

x	180	240	300	360	420	480	540	600
y								

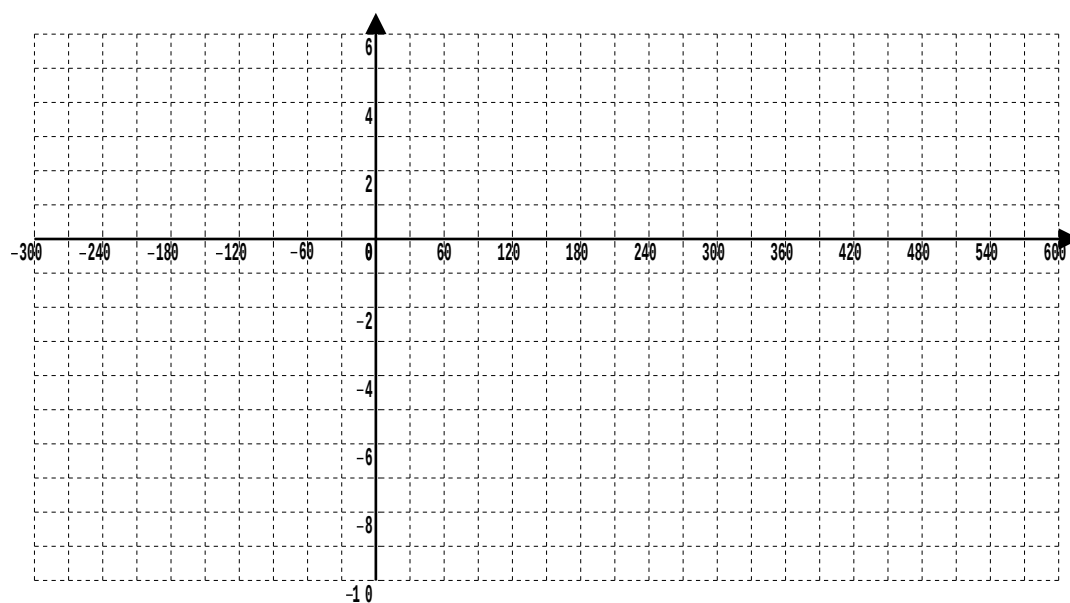
[3 marks]

(b) (i) On the grid below plot the graph of $y = f(x)$



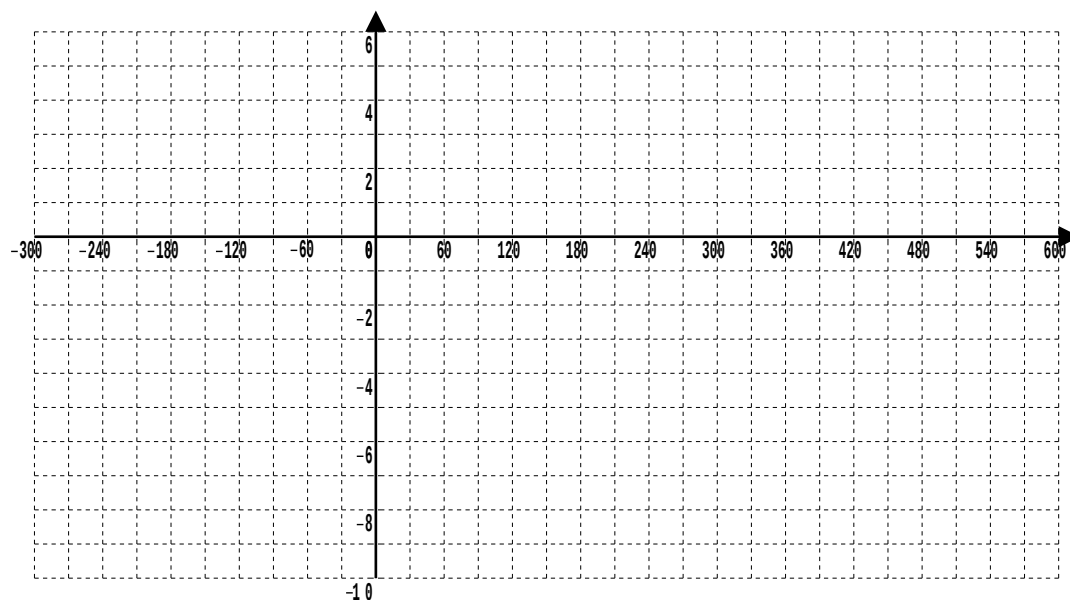
[2 marks]

(ii) On the grid below plot the graph of $y = | f(x) |$



[2 marks]

(iii) On the grid below plot the graph of $y = f(|x|)$



[2 marks]

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