Lesson 6

6.1 Composite Functions

The topic of Composite Functions is covered at GCSE.
At that level, the emphasis is on numerical problems, with the algebraic treatment kept simple and straightforward. For A Level, the emphasis is on the algebra along with concern about domains and ranges.

Keep in mind that $fg(x)$ means *eff the already gee'd x*

6.2 Example

Let s and t be the functions

$$s(x) = (x+1)^2 \qquad x \in \mathbb{R}$$

$$t(x) = x - 2 \qquad x \in \mathbb{R}$$

(a) Determine each of the following,

(i) $st(8)$ **(ii)** $ts(8)$

(iii) $ts(x)$ **(iv)** $st(x)$

[4 marks]

(b) Find the unique value of x such that

$$ts(x) = st(x)$$

[2 marks]

Notice that, in general, $st(x) \neq ts(x)$

6.3 Exercise

Any solution based entirely on graphical or numerical methods is not acceptable

Marks Available: 60

Question 1

Given that, $f(x) = 3x + 2, \quad x \in \mathbb{R}$ and $g(x) = 5x - 4, \quad x \in \mathbb{R}$

Find an expression for $gf(x)$ that does not contain any brackets.

[2 marks]

Question 2

Given that, $f(x) = 7x - 5, \quad x \in \mathbb{R}$ and $g(x) = 10 - x, \quad x \in \mathbb{R}$

Find an expression for $fg(x)$ that does not contain any brackets.

[2 marks]

Question 3

Given that, $f(x) = x^2 + x, \quad x \in \mathbb{R}$ and $g(x) = x + 3, \quad x \in \mathbb{R}$

(i) Find an expression for $fg(x)$ that does not contain any brackets.

[2 marks]

(ii) Find the two values of x such that, $fg(x) = 0$

[2 marks]

Question 4

Given that, $f(x) = 4x - 1, \quad x \in \mathbb{R}$ and $g(x) = x^2 + 1, \quad x \in \mathbb{R}$

Determine the following, giving answers free of any brackets;

(i) $f(3)$

(ii) $g(4)$

(iii) $fg(1)$

(iv) $gf(2)$

(v) $fg(x)$

(vi) State the range of your part **(v)** composite function

[1, 1, 2, 2, 3, 1 marks]

Question 5

$$f(x) = \frac{12}{x + 5} \qquad x \in \mathbb{R}, \; x \neq -5$$

$$g(x) = 6x - 5 \qquad x \in \mathbb{R}$$

Find a simplified expression for $fg(x)$ that does not contain any brackets.

[2 marks]

Question 6

$$s(x) = x^2 + x, \qquad x \in \mathbb{R}$$

$$t(x) = x - 2, \qquad x \in \mathbb{R}$$

(a) Determine;

(i) $s(5)$ **(ii)** $sss(1)$ **(iii)** $ttt(1)$

(iv) $ts(10)$ **(v)** $st(3)$ **(vi)** $ts(-1)$

[3 marks]

(b) Determine $s\,t\,(x)$ writing your answer without any brackets.

[2 marks]

(c) Find the two values of x for which $st(x) = 0$

[2 marks]

Question 7

$$f(x) = \sqrt{x} + 3 \qquad x \in \mathbb{R},\ x \geqslant 0$$

$$g(x) = x + 2 \qquad x \in \mathbb{R}$$

(i) Find $fg(x)$

[2 marks]

(ii) State the domain of $fg(x)$ given that it should be as large as possible

[1 mark]

(iii) State the corresponding range of $fg(x)$

[1 mark]

Question 8

$$f(x) = x^2 - 1 \qquad x \in \mathbb{R}$$

Solve the equation,

$$ff(x) = 0$$

[4 marks]

Question 9

$$f(x) = x^2 + 8 \qquad x \in \mathbb{R}$$

$$g(x) = 2x - 5 \qquad x \in \mathbb{R}$$

Solve the equation

$$fg(x) = gf(x)$$

giving your solutions as exact surds.

[5 marks]

Question 10

Let two functions, *m* and *n*, be;

$$m(x) = 10x - 9, \qquad x \in \mathbb{R}, \qquad x \geqslant 0.9$$

$$n(x) = 100 - \sqrt{x} \qquad x \in \mathbb{R}, \qquad 0 \leqslant x \leqslant 9820.81$$

Determine each of the following, giving bracket free answers;

(i) $n(64)$

(ii) $m(3)$

(iii) $mn(9)$

(iv) $nm(9)$

(v) $mn(x)$

(vi) $nm(x)$

[8 marks]

(vii) Function *m* is only defined on domain $x \geqslant 0.9$

To see why, calculate $m(0)$ then $nm(0)$

[2 marks]

(viii) Having had to restrict the domain of *m* so that *nm* exists, the domain of *n* has to then also be restricted so that its output can be fed into *m*. Calculate, $n(9820.8)$ and explain why any input greater than the 9820.81 would cause a problem.

[2 marks]

Observation : *fg*, can be formed only if the range of *g* is a subset of the domain of *f*.

Question 11

Let m and n be the functions;

$$m(x) = 9x - 5 \qquad x \in \mathbb{R}$$

$$n(x) = \sqrt{x - 7} \qquad x \in \mathbb{R},\ x \geqslant 7,$$

Evaluate each of the following;

(i) $mn(8)$ **(ii)** $mn(56)$

(iii) $mn\left(z^2 + 7\right)$ **(iv)** $mn\left(4z^2 + 7\right)$

[8 marks]