

Lesson 7

A-Level Pure Mathematics, Year 2

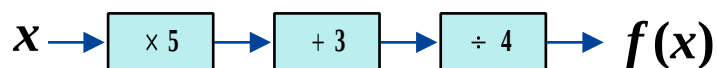
Functions II

7.1 Inverse Functions

Inverse functions have been met before at GCSE level (See Functions I).

Here is a question by way of recalling some of that previous knowledge;

- (i) Write down the function described by the following diagram,



[2 marks]

- (ii) Write down the inverse function $f^{-1}(x)$

[2 marks]

7.2 A More Advanced Example

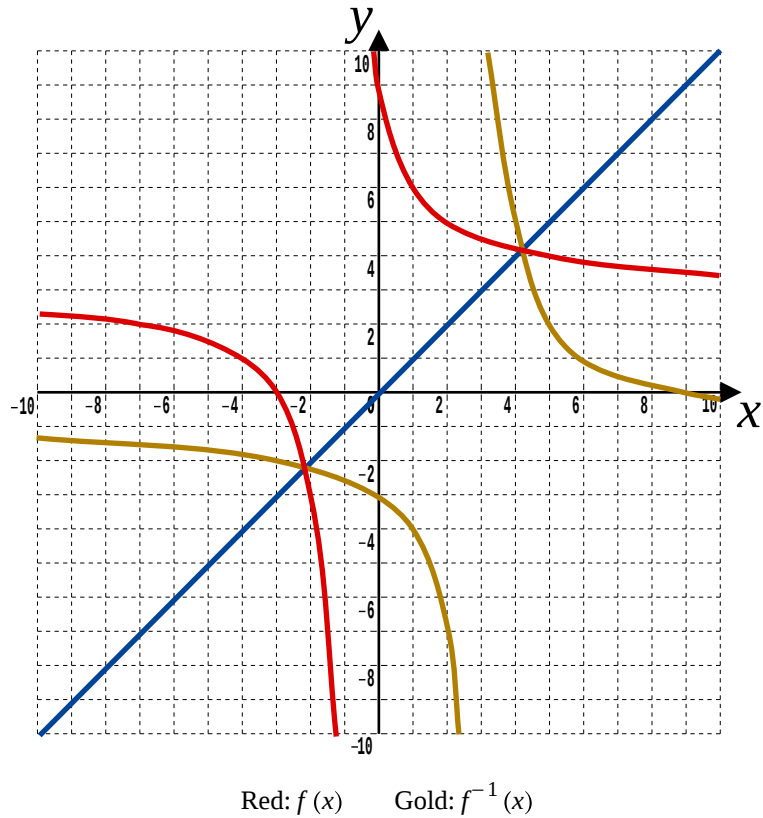
For the one-to-one function,

$$f(x) = \frac{6}{x+1} + 3$$

find $f^{-1}(x)$

[3 marks]

7.3 Inverse Graphically



(a) For the function, $f(x) = \frac{6}{x+1} + 3$

(i) Write down the domain:

[1 mark]

(ii) Write down the range:

[1 mark]

(b) For the inverse function,

$$f^{-1}(x) = \frac{6}{x-3} - 1$$

(i) Write down the domain:

[1 mark]

(ii) Write down the range:

[1 mark]

General Observations

- Geometrically, the inverse of a function is a reflection in the line $y = x$
- The domain of the function became the range of the inverse
- The range of the function became the domain of the inverse

Graphing the Function

The function, $f(x) = \frac{6}{x+1} + 3$ is better thought of as, $(y-3) = \frac{6}{(x+1)}$,

because the relationship to the graph of inverse proportion, $y = \frac{1}{x}$, is then obvious.

Focussing on the two asymptotes, one along the x -axis, the other along the y , it's then deduced that replacing x with $(x+1)$ will translate the y -axis asymptote to $x = -1$ and that replacing the y with $(y-3)$ will translate the x -axis asymptote to $y = 3$

Keeping in mind the fact that, for example,

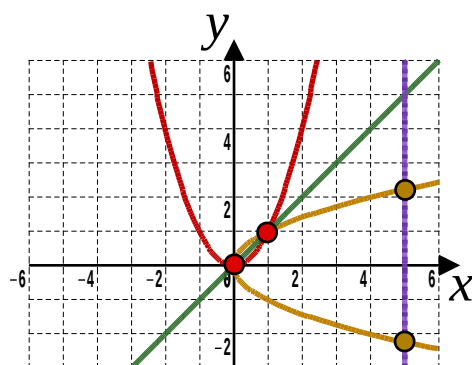
$y = \frac{1}{x}$, $y = \frac{6}{x}$, $y = \frac{12}{x}$, and indeed $y = \frac{k}{x}$ where k is a constant,

all have asymptotes along the x and y axis explains why the 6 has no effect on where the asymptotes of the transformed graph lie.

7.4 Existence of an Inverse

To have an inverse, a function must be one-to-one

The graph below shows (in red) the many-to-one function $f(x) = x^2$, $x \in \mathbb{R}$ and also (in gold) its reflection in the line $y = x$. However the reflection is not a function as a vertical line can be drawn that cuts it more than once.



If the inverse of a many-to-one function is sought, it must be broken up into pieces each of which is one-to-one and then, separately, the inverse of each piece found.

So, for example, to find the inverse of the many-to-one function $f(x) = x^2$ requires breaking it into two separate pieces,

$$g(x) = x^2, \quad x \geq 0$$

$$h(x) = x^2 \quad x < 0$$

and separately finding the inverse of each piece;

$$g^{-1}(x) = \sqrt{x} \quad x \geq 0$$

$$h^{-1}(x) = -\sqrt{x} \quad x \geq 0$$

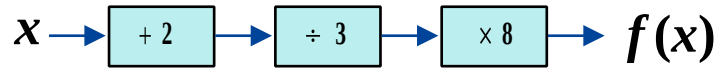
7.5 Exercise

Any solution based entirely on graphical or numerical methods is not acceptable

Marks Available: 50

Question 1

Consider the following diagram,



- (a) Write down the function described by the diagram

[1 mark]

- (b) Find the inverse function in two different ways,

- (i) By looking at the diagram and working through it 'backwards'
(As in section 7.1)

[2 marks]

- (ii) By algebraic manipulation.
(As in section 7.2)

[2 marks]

Question 2

Using a method of your choice, find the inverse of the function

$$g(x) = 8x + 3$$

[2 marks]

Question 3

Using a method of your choice, find the inverse of the function

$$h(x) = \frac{x}{5} - 4$$

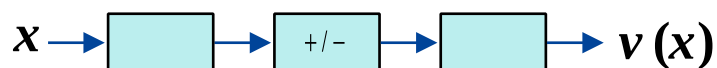
[2 marks]

Question 4

The function $v(x)$ is described by,

$$v(x) = 11 - 6x$$

A flow diagram for the function, shown below, includes the “change sign” instruction, which reverses the sign of whatever is fed into it.



- (a) Complete the flow diagram by filling in the two empty boxes with appropriate instructions.

[2 marks]

- (b) Find the inverse function in two different ways,

- (i) By looking at the diagram and working through it 'backwards'
(As in section 7.1)

[2 marks]

- (ii) By algebraic manipulation
(As in section 7.2)

[2 marks]

Question 5

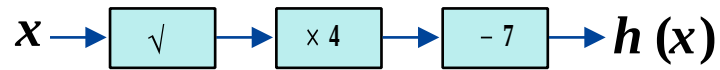
Using a method of your choice, find the inverse of the function

$$u(x) = \frac{3x}{5} + 4$$

[3 marks]

Question 6

Consider the following diagram,



- (a) (i) Write down the function described by the diagram

[1 mark]

- (ii) What is the domain of the function ?

[1 mark]

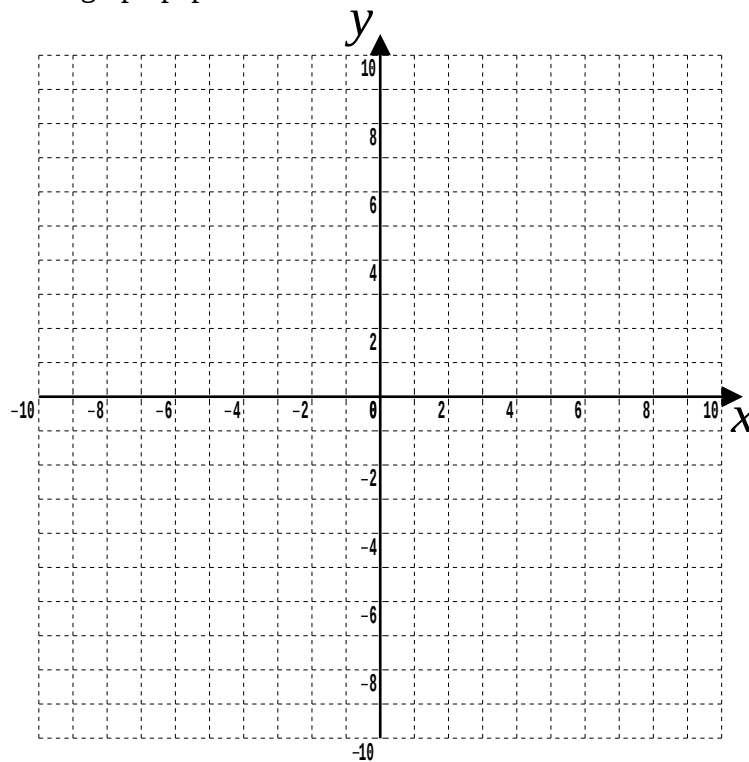
- (b) Find the inverse function.

[3 marks]

- (c) What is the domain of the inverse function ?

[1 mark]

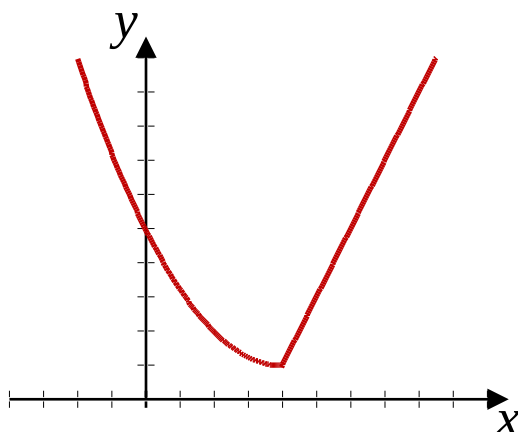
- (d) Provide a fairly accurate plot of both the original function and the inverse on the graph paper below.



[4 marks]

Question 7

A-Level Examination Question from June 2019, Paper 2, Q6 (Edexcel)



The diagram shows a sketch of $y = g(x)$, where,

$$g(x) = \begin{cases} (x - 2)^2 + 1 & x \leq 2 \\ 4x - 7 & x > 2 \end{cases}$$

(a) Find the value of $gg(0)$

[2 marks]

(b) Find all values of x for which $g(x) > 28$

[4 marks]

The function h is defined by, $h(x) = (x - 2)^2 + 1$, $x \leq 2$

(c) Explain why h has an inverse but g does not.

[1 mark]

(d) Solve the equation $h^{-1}(x) = -\frac{1}{2}$

[3 marks]

Question 8

A-Level Examination Question from June 2018, Paper C34, Q5 (Edexcel)

(i) The functions f and g are defined by

$$f : x \rightarrow e^{2x} - 5, \quad x \in \mathbb{R}$$

$$g : x \rightarrow \ln(3x - 1), \quad x \in \mathbb{R}, x > \frac{1}{3}$$

(a) Find f^{-1} and state its domain.

[3 marks]

(b) Find $fg(3)$, giving your answer in its simplest form.

[2 marks]

(ii) (a) Sketch the graph with equation $y = |4x - a|$ where a is a positive constant. State the coordinates of each point where the graph cuts or meets the coordinate axes.

[2 marks]

Given that, $|4x - a| = 9a$, where a is a positive constant,

(b) find the possible values of,

$$|x - 6a| + 3|x|$$

giving your answers, in terms of a , in their simplest form.

[5 marks]

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