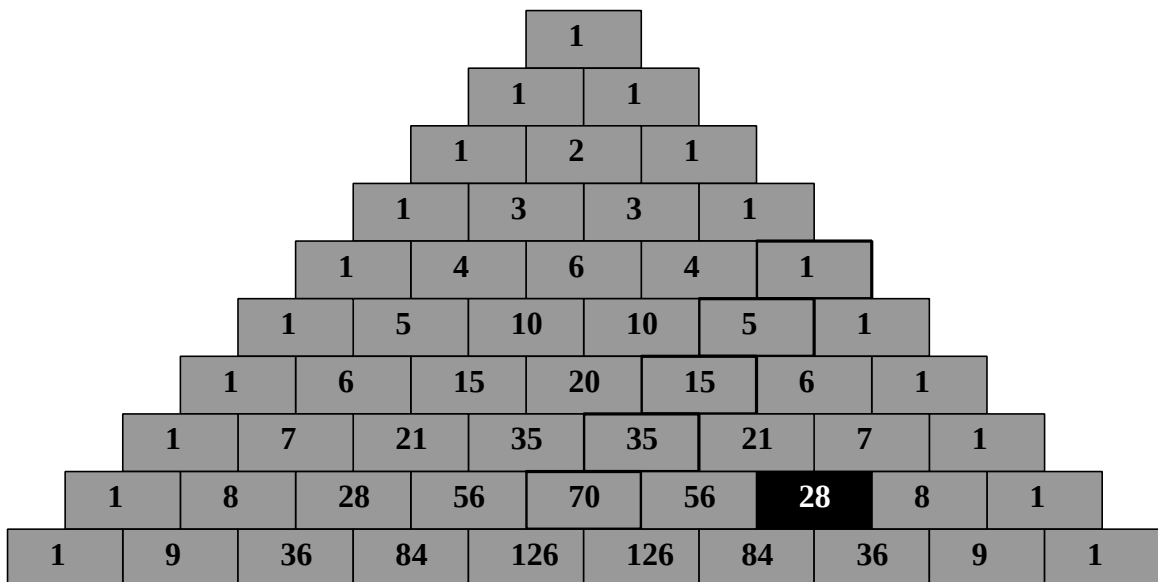


A-Level Pure Mathematics
Year 2

THE BINOMIAL THEOREM REVISITED



Lesson 1

A-Level Pure : The Binomial Theorem : Year 2

1.1 What Was Covered In Year 1

We looked at Binomial Expansion in Year 1.

Before taking the topic further, let's recall some of the techniques employed.

1.2 Example

(i) Use Pascal's triangle to expand the brackets;

$$(2 + 3x)^4$$

Give each term in your expansion in as simple a form as possible.

$$\begin{aligned}(2 + 3x)^4 &= \binom{4}{0} \times \binom{3}{0} \times \binom{2}{0} \times \binom{1}{0} \\&\quad + \binom{4}{1} \times \binom{3}{1} \times \binom{2}{1} \times \binom{1}{1} \\&\quad + \binom{4}{2} \times \binom{3}{2} \times \binom{2}{2} \times \binom{1}{2} \\&\quad + \binom{4}{3} \times \binom{3}{3} \times \binom{2}{3} \times \binom{1}{3} \\&\quad + \binom{4}{4} \times \binom{3}{4} \times \binom{2}{4} \times \binom{1}{4}\end{aligned}$$

(ii) By using your part (i) answer, or otherwise, expand the brackets of;

$$(2 + 3\sqrt{x})^4$$

1.3 Exercise

Question 1

(i) Use Pascal's triangle to expand the brackets;

$$(5 - 2x)^3$$

Give each term in your expansion in as simple a form as possible.

The following may be of help in setting out your working....

$$\begin{aligned}(5 - 2x)^3 &= \begin{pmatrix} \\ \\ \end{pmatrix} \times \begin{pmatrix} \\ \\ \end{pmatrix} \times \begin{pmatrix} \\ \\ \end{pmatrix} \\ &+ \begin{pmatrix} \\ \\ \end{pmatrix} \times \begin{pmatrix} \\ \\ \end{pmatrix} \times \begin{pmatrix} \\ \\ \end{pmatrix} \\ &+ \begin{pmatrix} \\ \\ \end{pmatrix} \times \begin{pmatrix} \\ \\ \end{pmatrix} \times \begin{pmatrix} \\ \\ \end{pmatrix} \\ &+ \begin{pmatrix} \\ \\ \end{pmatrix} \times \begin{pmatrix} \\ \\ \end{pmatrix} \times \begin{pmatrix} \\ \\ \end{pmatrix}\end{aligned}$$

(ii) By using your part (i) answer, or otherwise, expand the brackets of;

$$(5 - 2\sqrt{x})^3$$

Question 2

Expand the brackets

(i)

$$(11 - x)^4$$

(ii)

$$(10 + x^4)^5$$

Question 3

- (i) Expand the brackets, simplifying each term;

$$\left(x + \frac{5}{x} \right)^4$$

- (ii) Which is the term that is independent in x ?

- (iii) What is the coefficient on the term in x^2 ?

Question 4

(i) Expand the brackets

$$\left(1 + \frac{x}{2} \right)^4$$

In Year 2, it becomes crucial to first adjust the question so that the bracket contains the number 1 as its first term.

(ii) Prove that,

$$(4 + 2x)^4 = 256 \left(1 + \frac{x}{2} \right)^4$$

(iii) Hence, using your part (i) answer, expand the following;

$$(4 + 2x)^4$$

Question 5

(i) Given that,

$$(6 + 9 x^2)^5 = k (1 + a x^2)^5$$

State the value of k . and the value of a .

(ii) Expand the following in ascending powers of x , where a has the value determined in part (i)

$$(1 + a x^2)^5$$

(iii) Use your previous answers to expand the brackets of

$$(6 + 9 x^2)^5$$

Question 6

By using Pascal's Triangle, or otherwise, expand the brackets;

$$\left(\frac{1}{3x^2} + x^3 \right)^4$$