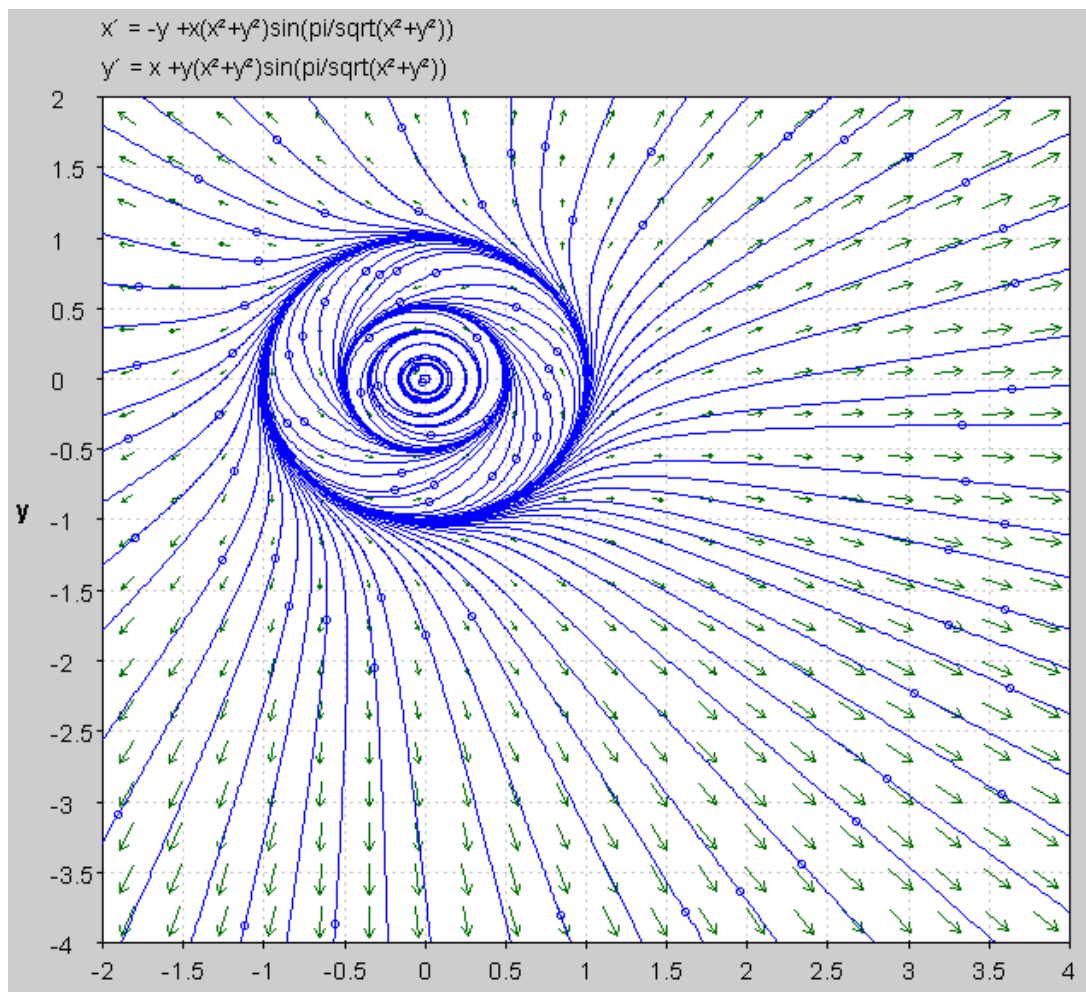


A-Level Pure Mathematics
Year 2

DIFFERENTIAL EQUATIONS I



Differential Equations I

Lesson 1

A-Level Pure Mathematics : Year 2

Differential Equations I

1.1 Type One

Three types of differential equation are to be considered starting in this lesson with the most straight forward.

A Type One differential equation is of the form

$$\frac{dy}{dx} = f(x)$$

These have been encountered before, in the Year 1 pure mathematics course.

Example

A-Level Examination Question from January 2013, Paper C1, Q8 (Edexcel)

$$\frac{dy}{dx} = -x^3 + \frac{4x - 5}{2x^3}, \quad x \neq 0$$

Given that $y = 7$ at $x = 1$, find y in terms of x , giving each term in its simplest form.

Teaching Video : [http://www.NumberWonder.co.uk/Video/v9066\(1\).mp4](http://www.NumberWonder.co.uk/Video/v9066(1).mp4)

[6 marks]

The Year 2 version of such Type One questions will draw on the fact that it is now known how to integrate many more functions, using many more techniques.

1.2 Exercise

*Any solution based entirely on graphical
or numerical methods is not acceptable*

Marks Available : 50

Question 1

Solve the differential equation,

$$\frac{dy}{dx} = \cos x \quad \text{given that } y = 0.5 \text{ when } x = \frac{\pi}{2}$$

[4 marks]

Question 2

Solve the differential equation

$$\frac{dy}{dx} = \sec^2 x \quad \text{given that } y = 1 \text{ when } x = \frac{3\pi}{4}$$

[5 marks]

Question 3

- (i) Find the displacement, s cm, from O of a particle at time t s, if its velocity, v cm.s⁻¹, is given by the differential equation

$$v = \frac{ds}{dt} = t^2 - t + 4$$

and the displacement is 100 cm at time 6 s.

[3 marks]

- (ii) What will be the particle's displacement when $t = 3$ s ?

[2 marks]

Question 4

Solve the differential equation,

$$\frac{dy}{dx} = x^2 (x^3 + 5)^4 \text{ given that } y = 209 \text{ when } x = 0$$

Hint : Chain rule backwards.

[5 marks]

Question 5

- (i) Find the general solution of the differential equation

$$5x \frac{dy}{dx} - 1 = 0$$

[4 marks]

- (ii) Given that $y = 0$ when $x = 3$, find the particular solution.
Give your answer in an elegant a form as possible.

[3 marks]

Question 6

- (i) Find the general solution of the differential equation

$$\sec x \frac{dy}{dx} - x = 0$$

Hint : Integration by parts.

[4 marks]

- (ii) Given that $y = \pi$ when $x = \frac{\pi}{2}$, find the particular solution.

[3 marks]

Question 7

Solve the differential equation,

$$\frac{dy}{dx} = \frac{1}{9 + x^2} \quad \text{given that } y = \frac{\pi}{9} \text{ when } x = \sqrt{3}$$

Hint : Let $x = 3 \tan u$

[8 marks]

Question 8

- (i) Find the displacement s m of a particle t s after leaving O , where

$$t \frac{ds}{dt} = t^2 + 4$$

[5 marks]

- (ii) Given that $s = 4 \ln 2$ when $t = 2$, and $s = a + b \ln 2$ when $t = 4$, find a and b .

[4 marks]

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Teachers may obtain detailed worked solutions to the exercises by email from MHHShrewsbury@Gmail.com