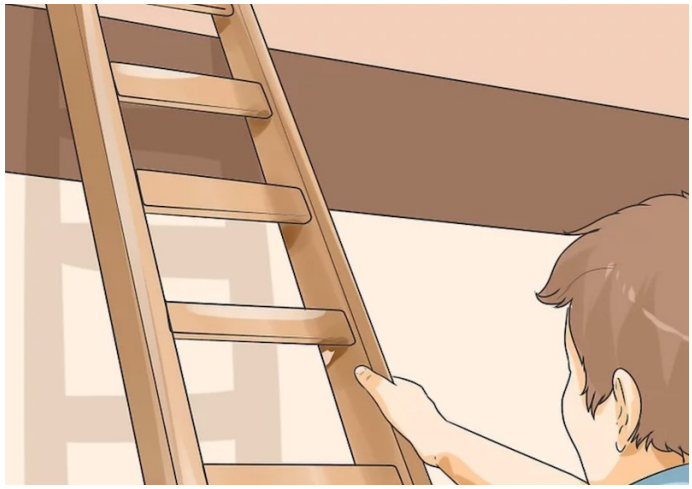
Lesson 3

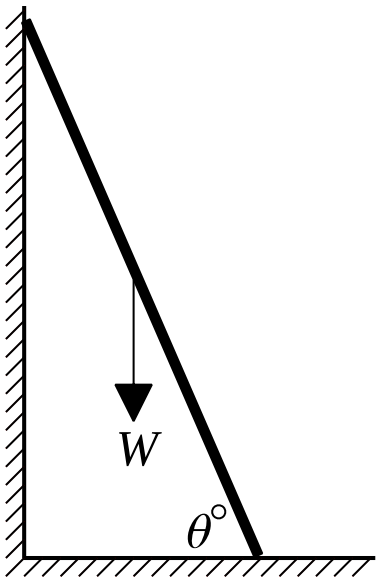
A-Level Applied Mathematics
Mechanics : Moments II : Year 2

3.1 The Non Horizontal Rod

In the previous lessons, the problems considered involved moments in which some of the forces acting on the horizontal rod where not at right angles to the rod. In this lesson the forces will all be either horizontal or vertical but the rod itself will no longer be horizontal. The classic problem in this regard is that of a ladder resting against a wall. Earlier work with friction is called back into play, as it is friction between the foot of a ladder and the ground that allows the ladder to remain in position.

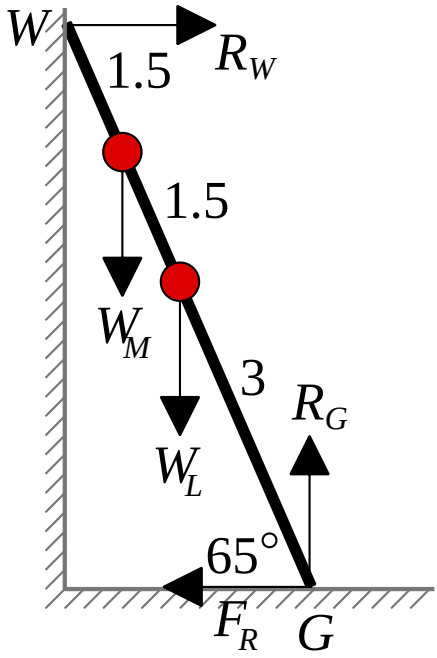


3.2 The Ladder and The Wall



When thinking about a ladder resting against a wall, there is a resemblance to the problem of a block of weight W on a slope at $\theta°$ to the horizontal. So one method of analysis would be to resolve the weight of the ladder, W, into component parts. The component parallel the slope would be $W \sin \theta$ and perpendicular to the slope would be $W \cos \theta$. However, in many ladder problems this is often not the best way to proceed. To see why this is so a typical example will be considered in some depth.

3.3 Example (The Question)



A ladder *WG* of weight W_L = 120 N and length 6 metres has one end *G* resting on rough horizontal ground. The other end *W* rests against a smooth vertical wall. A man of weight W_M = 750 N has gone three-quarters of the way up the ladder but has stopped because he can sense that the ladder is about to slip.
Find the coefficient of friction, μ, between the foot of the ladder and the ground given that the ladder is inclined at 65° to the horizontal.

3.4 Example (The Discussion)

Before trying to solve the problem, a discussion of the diagram above is helpful.
The wall is described as “smooth” which is why the reaction where the ladder rests against the wall is perpendicular to the wall.
It is because the question has asked for the coefficient of friction, μ, with the ground that the foot of the ladder has a normal reaction, R_G, and a friction force, F_R. The intention is to determine R_G (which is perpendicular to the ground) and F_R (which is parallel to the ground) and then obtain μ using,

$$F_R = \mu R$$

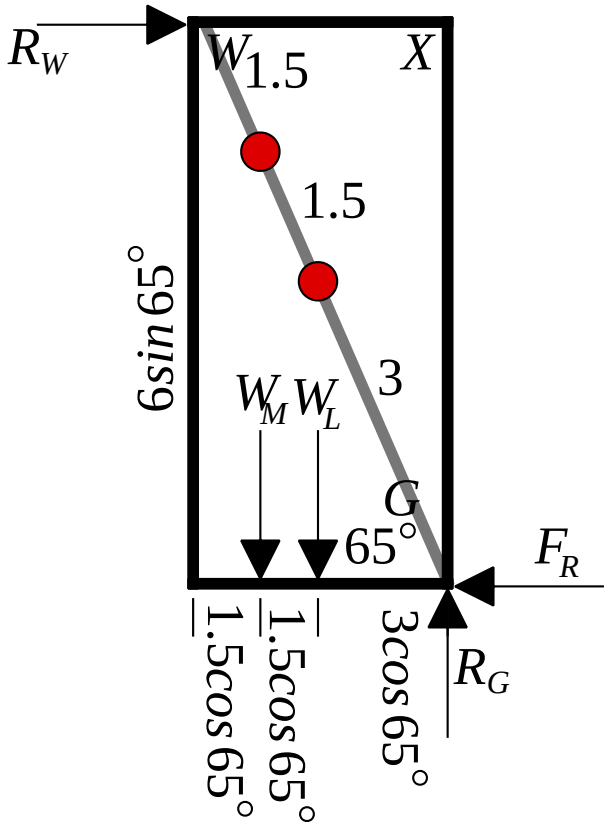
A different question could have asked for the overall action of the ground on the ladder, as the questions in Lesson 1 did. If this were asked for, having already found R_G and F_R, by the method about to be described, this could then be found easily using,

$R_A = \sqrt{(R_G)^2 + (F_R)^2}$ and $\tan\theta = \dfrac{R_G}{F_R}$ where θ is angle to the horizontal.

An important fact to notice is that all of the forces in this question are naturally either vertical or horizontal. This makes resolving every single force into components that are parallel or perpendicular to the plane an uneccessarily complicated way to do the question. Instead there is a really clever technique, that will be developed next, where the rod itself is replaced by an equivalent structure.

3.5 Example (The Solution)

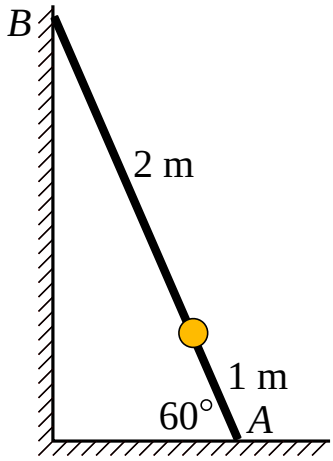
The ladder is replaced by a rigid rectangle shaped light and inextensible rod, one side of which lies along the wall and another along the ground. Each force is projected along its line of action to act on the rectangular rod. Like the ladder from whence it came, The rectangular rod is in equilibrium. So, the sum of moments about any pivot is zero, and the resultant force in any given direction (especially horizontally or vertically) is zero. Here is the new diagram with distances between key points added,



Teaching Video : http://www.NumberWonder.co.uk/Video/v9084(3).mp4

3.6 Exercise

Question 1



A ladder *AB*, of mass *m* and length 3 metres, has one end *A* resting on rough horizontal ground. The other end *B* rests against a smooth vertical wall. A load of mass 2*m* is fixed on the ladder at the point *C*, where *AC* = 1 metre.

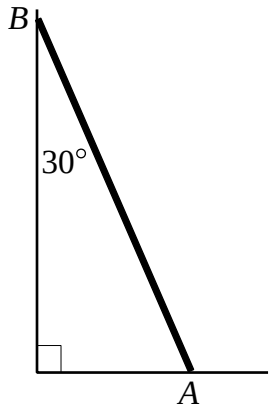
The ladder is modelled as a uniform rod in a vertical plane perpendicular to the wall and the load is modelled as a particle. The ladder rests in limiting equilibrium at an angle of 60° with the ground.

(i) Draw a force diagram in which the ladder has been replaced with a rigid rectangular rod which has one edge along the ground and another along the wall. Mark on all significant forces, projected onto the rectangular rod, and use simple trigonometry to obtain key distances between the forces on the sides of the rectangular rod.

(ii) Find the coefficient of friction between the ladder and the ground.

Question 2

A-Level Examination Question from January 2017, IAL, M2, Q7



A uniform rod AB has mass m and length $2a$. The end A is in contact with rough horizontal ground and the end B is in contact with a smooth vertical wall. The rod rests in equilibrium in a vertical plane perpendicular to the wall and makes an angle of 30° with the wall, as shown. The coefficient of friction between the rod and the ground is μ.

(a) Find, in terms of m and g, the magnitude of the force exerted on the rod by the wall.

[4 marks]

(b) Show that $\mu \geqslant \frac{\sqrt{3}}{6}$

[3 marks]

A particle of mass km is now attached to the rod at B.

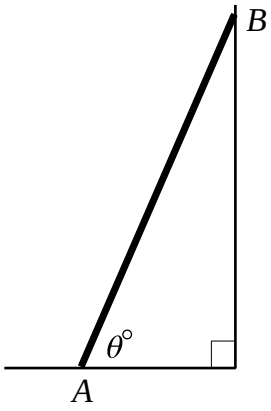
Given that $\mu = \frac{\sqrt{3}}{5}$ and that the rod in now in limiting equilibrium,

(c) find the value of k

[6 marks]

Question 3

A-Level Specimen Examination Question for the (Cancelled) June 2020 Paper 3, Q2



The ladder AB has length $2a$ and weight W. The ladder rests in equilibrium with end A on rough horizontal ground and end B against a smooth vertical wall.

The ladder rests in a vertical plane perpendicular to the wall. and is inclined at angle θ to the ground. The coefficient of friction between the ladder and the ground is μ.

The ladder is on the point of slipping. The ladder is modelled as a uniform rod.

Show that $\mu = \dfrac{1}{2\tan\theta}$

[7 marks]