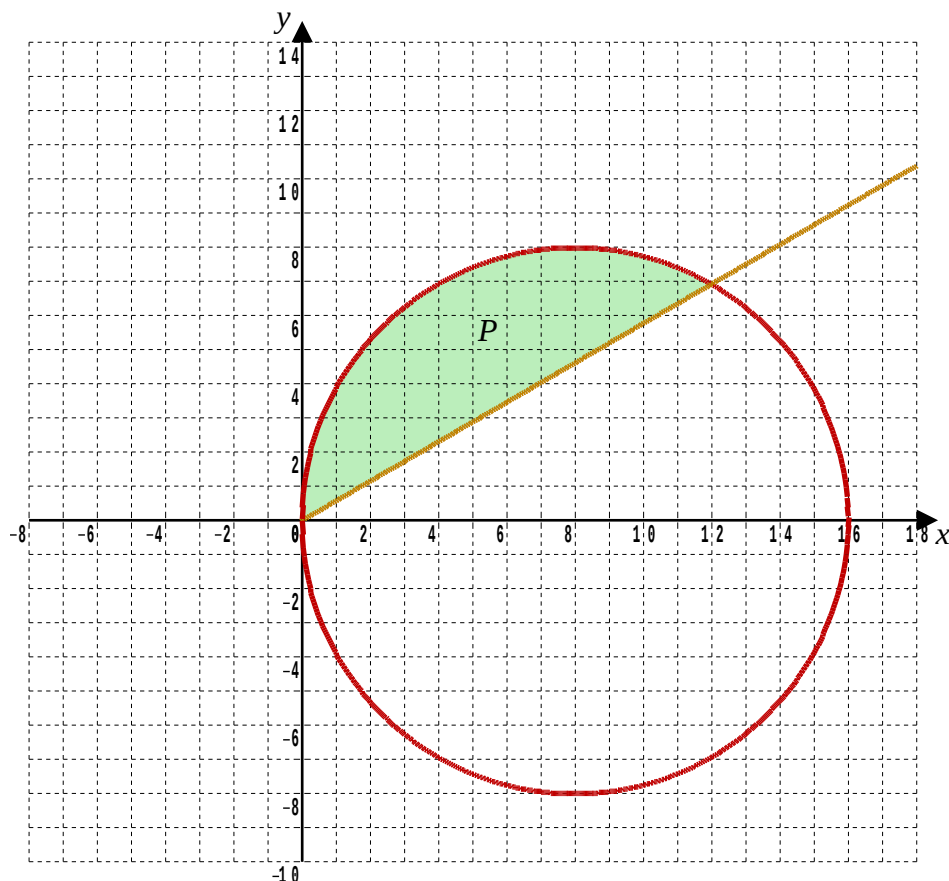


Lesson 7

Further A-Level Pure Mathematics, Core 2 Polar Coordinates

7.1 Area Problems, Taken Further

Here is a recap of the type of problem tackled in the previous lesson,



The segment shaded is in the circle $r = 16 \cos \theta$ and bounded by $\theta = \frac{\pi}{6}$

It's area is given by,

$$\begin{aligned} P &= \frac{1}{2} \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} 16^2 \cos^2 \theta \, d\theta \\ &= \frac{16^2}{2} \times \frac{1}{4} \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} (2 + 2 \cos 2\theta) \, d\theta \\ &= 32 \left[2\theta + \sin 2\theta \right]_{\frac{\pi}{6}}^{\frac{\pi}{2}} \\ &= 32 \left[\pi + \sin \pi - \frac{\pi}{3} - \sin \left(\frac{\pi}{3} \right) \right] \\ &= 32 \left[\pi + 0 - \frac{\pi}{3} - \frac{\sqrt{3}}{2} \right] \\ &= \frac{64\pi}{3} - 16\sqrt{3} \quad (\text{About } 39.3) \end{aligned}$$

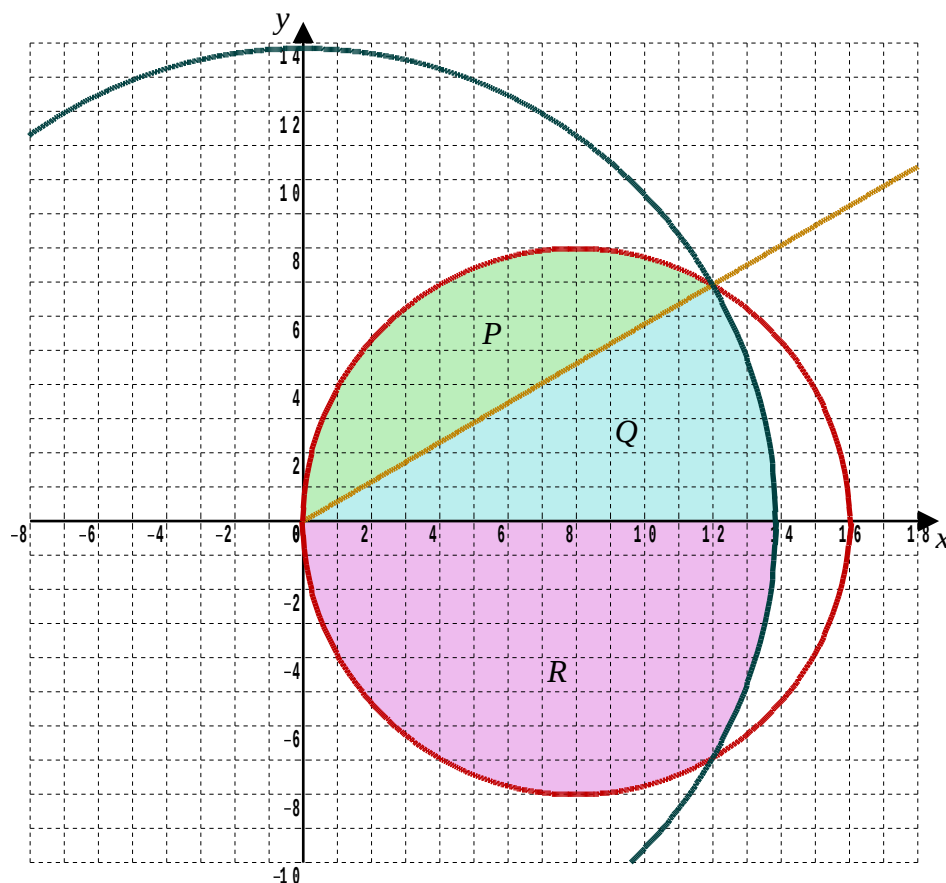
With this in mind, a related problem is considered next.

7.2 Area of Overlap

Find the area of the overlap between the two circles with polar equations,

$$r = 16 \cos \theta \text{ and } r = \sqrt{192}$$

This area is the total areas of the green, blue and red shaded regions below,



To find where the circles intersect,

$$16 \cos \theta = \sqrt{192} \Rightarrow \cos \theta = \frac{\sqrt{3}}{2} \Rightarrow \theta = \pm \frac{\pi}{6}$$

Thus the gold line $\theta = \frac{\pi}{6}$ is added to the graph.

Half of the area sought is the green area, P , plus the blue area, Q .

But the green area, P , is the same as that worked out in the previous problem.

$$\begin{aligned} Q &= \frac{1}{2} \int_0^{\frac{\pi}{6}} 192 \, d\theta \\ &= 16\pi \end{aligned}$$

$$\therefore \text{Area of overlap} = 2 \times (P + Q)$$

$$\begin{aligned} &= 2 \left(\frac{64\pi}{3} - 16\sqrt{3} + 16\pi \right) \\ &= \frac{224\pi}{3} - 32\sqrt{3} \quad (\text{About } 179) \end{aligned}$$

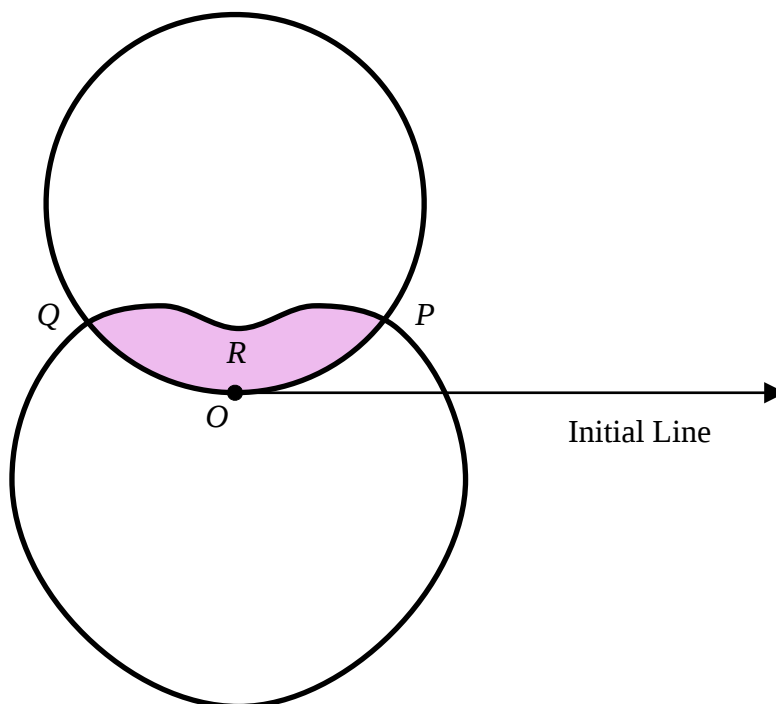
7.3 Exercise

*Any solution based entirely on graphical
or numerical methods is not acceptable*

Marks Available : 21

Question 1

Further A-Level Examination Question from June 2018, FP2, Q8 (Edexcel)



The sketch is of the curves with polar equations

$$r = 2 \sin \theta \quad 0 \leq \theta \leq \pi$$

$$r = 1.5 - \sin \theta \quad 0 \leq \theta \leq 2\pi$$

The curves intersect at the points P and Q

- (a) Find the polar coordinates of the point P and the polar coordinates of the point Q

[3 marks]

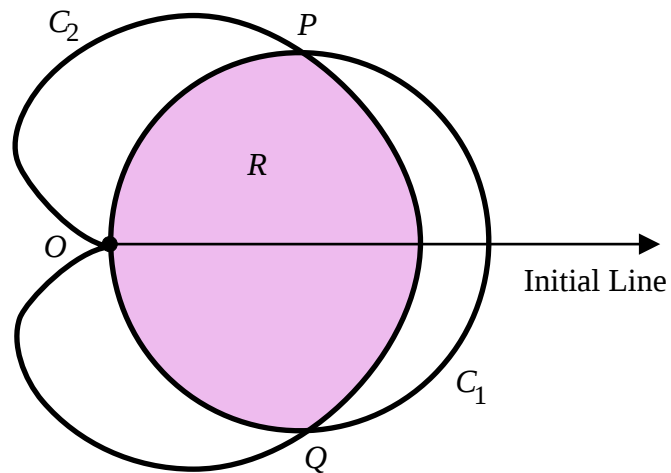
The region R , shown shaded, is enclosed by the two curves.

- (b)** Find the exact area of R giving your answer in the form $p\pi + q\sqrt{3}$, where p and q are rational numbers to be found.

[8 marks]

Question 2

Further A-Level Examination Question from June 2016, FP2, Q8 (Edexcel)



The sketch is of the curve C_1 with equation

$$r = 7 \cos \theta \quad -\frac{\pi}{2} < \theta \leq \frac{\pi}{2}$$

and the curve C_2 with equation

$$r = 3(1 + \cos \theta) \quad -\pi < \theta \leq \pi$$

The curves C_1 and C_2 both pass through the pole and intersect at the point P and at the point Q

(a) Find the polar coordinates of P and the polar coordinates of Q

[3 marks]

The regions enclosed by the curve C_1 and the curve C_2 overlap, and the common region is shaded in the sketch.

(b) Find the area of R

[7 marks]

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Teachers may obtain detailed worked solutions to the exercises by email from mhh@shrewsbury.org.uk