

Lesson 9

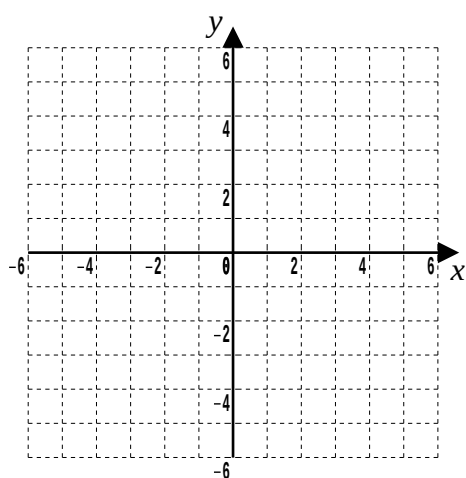
Further A-Level Pure Mathematics, Core 2 Polar Coordinates

9.1 Revision

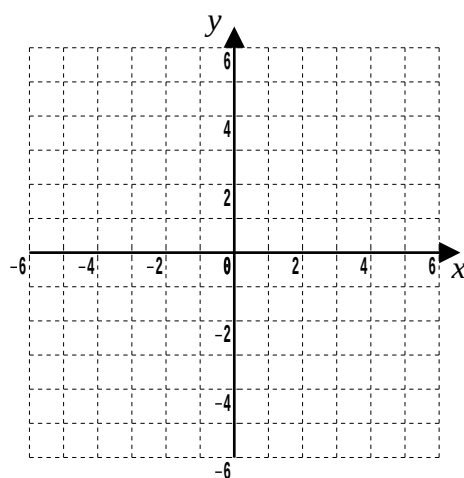
*Any solution based entirely on graphical
or numerical methods is not acceptable*
Marks Available : 40

Question 1

On each grid, sketch the graph of the polar equation given,

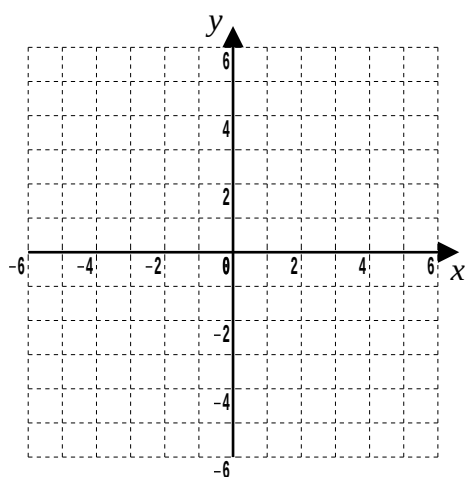


(i) $r = 3 \csc \theta$

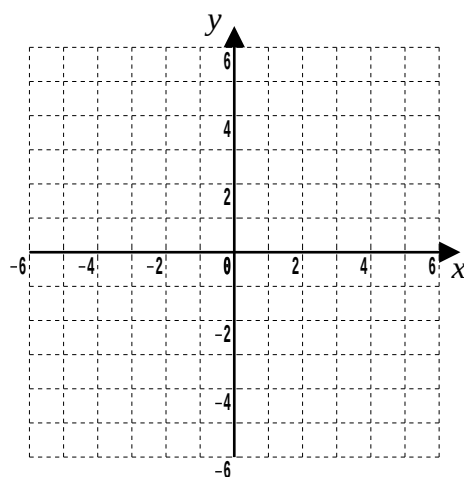


(ii) $r = 3 \cos \theta$

[1, 1 marks]



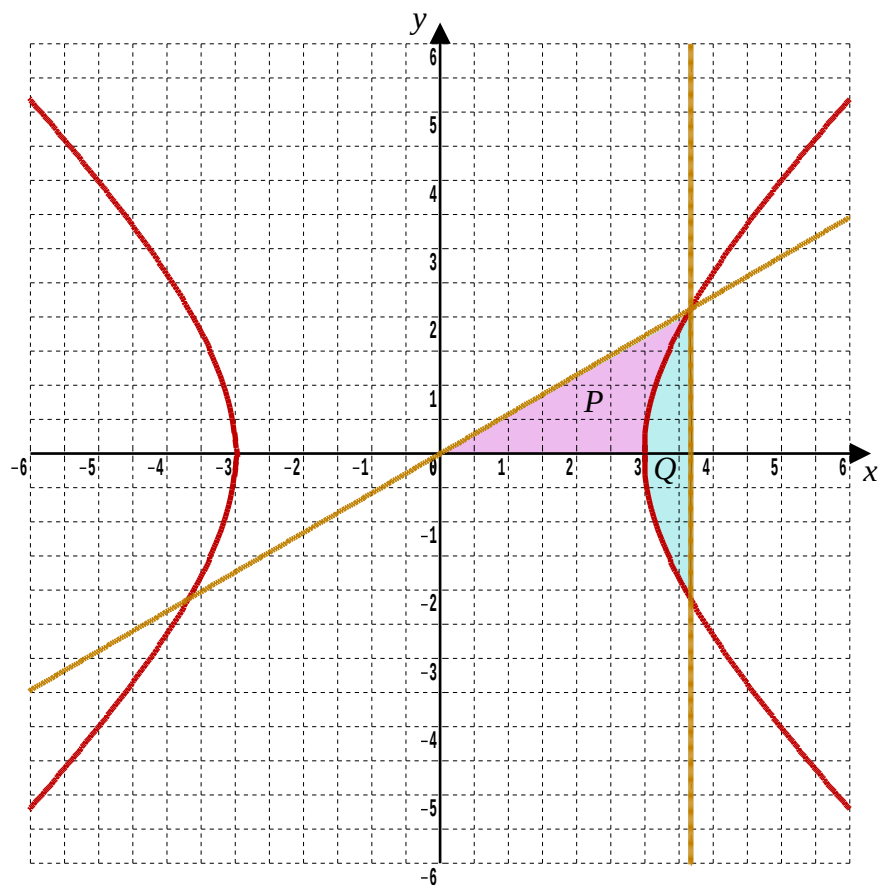
(iii) $r = \frac{2}{\cos \theta + 2 \sin \theta}$



(iv) $r = 4\sqrt{2} \sin (\theta - 45^\circ)$

[1, 1 marks]

Question 2



The curve in the graph is of the hyperbola with polar equation $r^2 = 9 \sec 2\theta$

Also shown is the vertical line with polar equation $r = \frac{3\sqrt{6}}{2} \sec \theta$

- (i) Show that the curve and vertical line intersect when $\theta = \pm \frac{\pi}{6}$

[4 marks]

(ii) The sloping straight line has polar equation $\theta = \frac{\pi}{6}$

Show that the area shaded and marked P is $\frac{9}{4} \ln(2 + \sqrt{3})$ by evaluating the following integral,

$$\text{Area } P = \frac{1}{2} \int_0^{\frac{\pi}{6}} 9 \sec 2\theta \, d\theta$$

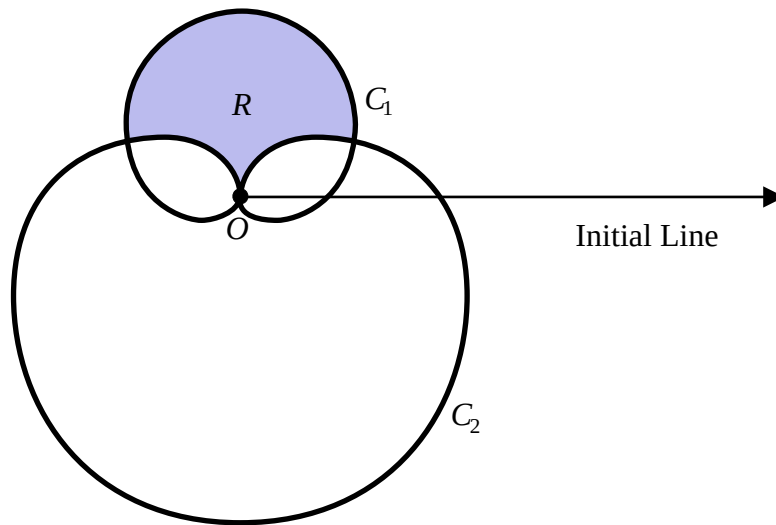
[5 marks]

(iii) Hence, or otherwise, find the exact area between the hyperbola and the vertical line which is shaded blue and marked Q .

[4 marks]

Question 3

Further A-Level Examination Question from October 2020, FP1, Q3 (Edexcel)



The sketch is of two curves C_1 and C_2 with polar equations,

$$C_1 : r = (1 + \sin \theta) \quad 0 \leq \theta < 2\pi$$

$$C_2 : r = 3(1 - \sin \theta) \quad 0 \leq \theta < 2\pi$$

The region R lies inside C_1 and outside C_2 and is shown shaded in the sketch.

Show the area of R is $p\sqrt{3} - q\pi$ where p and q are integers to be determined.

[9 marks]

Question 4

Further A-Level Examination Question from November 2021, FP2, Q6 (Edexcel)

The curve C has polar equation $r = a(p + 2 \cos \theta)$, $0 \leq \theta < 2\pi$

where a and p are positive constants and $p > 2$. There are exactly four points on C where the tangent is perpendicular to the initial line.

(a) Show that the range of possible values for p is $2 < p < 4$

[5 marks]

(b) Sketch the curve with equation $r = a(3 + 2 \cos \theta)$, $0 \leq \theta < 2\pi$
where $a > 0$

[1 mark]

John digs a hole in his garden in order to make a pond.

The pond has a uniform horizontal cross section that is modelled by the curve $r = 20(3 + 2 \cos \theta)$, $0 \leq \theta < 2\pi$, with r measured in centimetres.

The depth of the pond is 90 centimetres.

Water flows through a hosepipe into the pond at a rate of 50 litres per minute.

Given that the pond is initially empty,

(c) determine how long it will take to completely fill the pond with water using the hosepipe, according to the model.

Give your answer to the nearest minute.

[7 marks]

(d) State a limitation of the model.

[1 mark]

This document is a part of a **Mathematics Community Outreach Project** initiated by Shrewsbury School

It may be freely duplicated and distributed, unaltered, for non-profit educational use

In October 2020, Shrewsbury School was voted “**Independent School of the Year 2020**”

© 2025 Number Wonder

Teachers may obtain detailed worked solutions to the exercises by email from MHHShrewsbury@Gmail.com