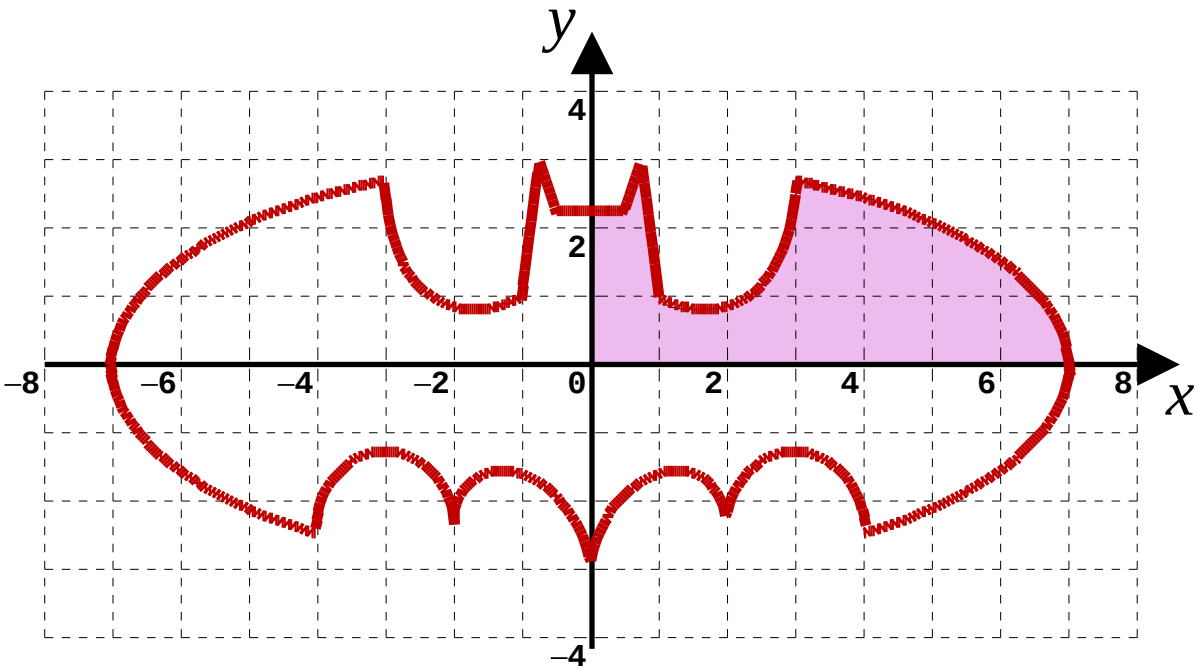
A-Level Pure Mathematics
Year 2

INTEGRATION III



"The Batman Curve"

$$\textit{Four curves base} \ : \ f(x) = \left|\frac{x}{2}\right| - \frac{3\sqrt{33}-7}{122}x^2 - 3 + \sqrt{1-(\,||x|-2|-1\,)^2}$$

$$\textit{Sides of face} \ : \ g(x) = 9\sqrt{\frac{|\,-(\,|x|-1\,)\,(\,|x|-0.75\,)\,|}{-(\,|x|-1\,)(\,|x|-0.75\,)}} - 8|x|$$

$$\textit{Ears} \ : \ h(x) = 3\,|x| + 0.75\sqrt{\frac{|\,-(\,|x|-0.5\,)\,(\,|x|-0.75\,)\,|}{-(\,|x|-0.5\,)(\,|x|-0.75\,)}}$$

$$\textit{Forehead} \ : \ p(x) = 2.25\sqrt{\frac{|\,-(x-0.5)\,(x+0.5)\,|}{-(x-0.5)(x+0.5)}}$$

$$\textit{Shoulders} \ : \ q(x) = \frac{6\sqrt{10}}{7} + (\,1.5 - 0.5\,|x|\,)\sqrt{\frac{||x|-1|}{|x|-1}} - \frac{6\sqrt{10}}{14}\sqrt{4-(\,|x|-1\,)^2}$$

$$\textit{Wings} \ : \ r(x) = 3\sqrt{1-\left(\frac{x}{7}\right)^2}\sqrt{\frac{||x|-3|}{|x|-3}} \qquad s(x) = -3\sqrt{1-\left(\frac{x}{7}\right)^2}\sqrt{\frac{||x|-4|}{|x|-4}}$$

Lesson 1

A-Level Pure Mathematics : Year 2

Integration III

1.1 Integration by Substitution

Example

A-Level Examination Question from January 2006, Paper C4, Q3 (Edexcel)

Using the substitution $u^2 = 2x - 1$, or otherwise, find the exact value of

$$\int_1^5 \frac{3x}{\sqrt{2x - 1}}\, dx$$

[8 marks]

1.2 Exercise

Any solution based entirely on graphical or numerical methods is not acceptable

Marks Available : 50

Question 1

Evaluate the following integral using the substitution given;

$$\int_3^{11} \frac{x}{\sqrt{x-2}}\,dx \qquad u^2 = x - 2$$

[8 marks]

Question 2

Evaluate the following integral using the substitution given;

$$\int_6^{10} \frac{x\,(x-4)}{(x-2)^2}\,dx \qquad u = x - 2$$

[8 marks]

Question 3

Evaluate exactly the following integral using the substitution given;

$$\int_2^6 \frac{x - 1}{\sqrt{2x - 3}}\, dx \qquad u^2 = 2x - 3$$

[8 marks]

Question 4

Evaluate the following integral using the substitution given;

$$\int_{0.5}^{1} \frac{1}{\sqrt{1 - x^2}}\, dx \qquad x = \sin\theta$$

Warning : You must use **RADIANS**

[8 marks]

Question 5

Evaluate the following integral using the substitution given;

$$\int_2^{2.5} \frac{2x + 1}{(3 - x)^6}\, dx \qquad u = 3 - x$$

[9 marks]

Question 6

Evaluate the following integral using the substitution given;

$$\int_{-0.25}^{0.5} \frac{48x}{(2x+1)^5}\,dx \qquad u = 2x + 1$$

[9 marks]