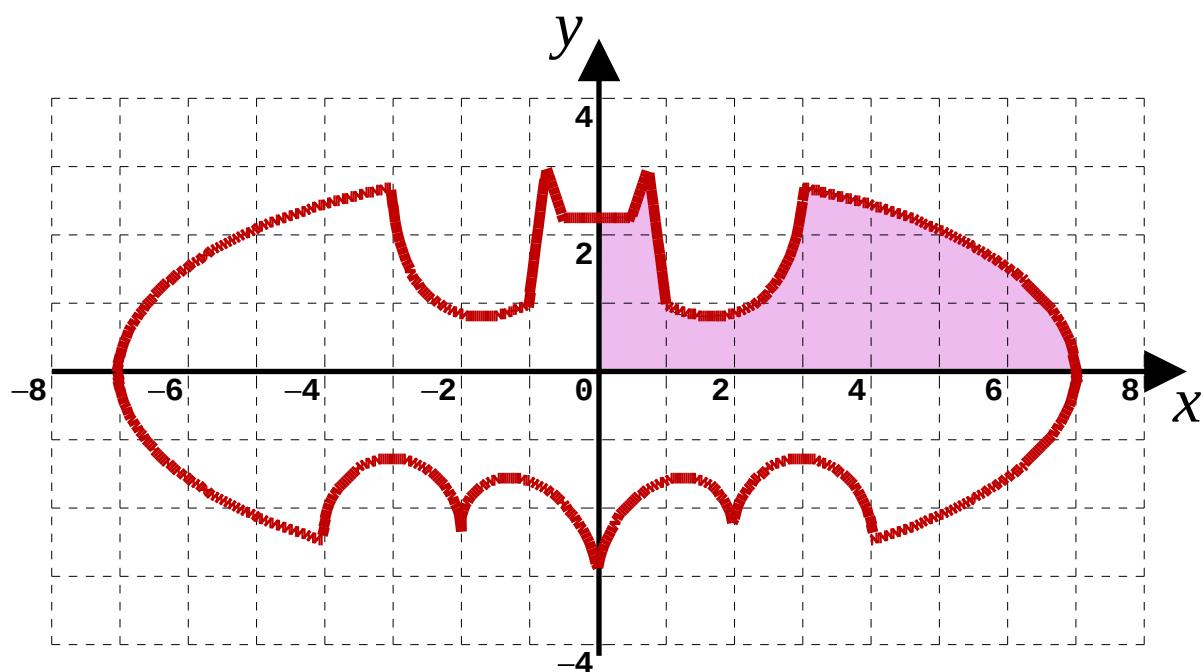


A-Level Pure Mathematics
Year 2

I N T E G R A T I O N

I I I



“The Batman Curve”

$$\text{Four curves base : } f(x) = \left| \frac{x}{2} \right| - \frac{3\sqrt{33} - 7}{122} x^2 - 3 + \sqrt{1 - (|x| - 2 - 1)^2}$$

$$\text{Sides of face : } g(x) = 9\sqrt{\frac{|1 - (|x| - 1)(|x| - 0.75)|}{-(|x| - 1)(|x| - 0.75)}} - 8|x|$$

$$\text{Ears : } h(x) = 3|x| + 0.75\sqrt{\frac{|1 - (|x| - 0.5)(|x| - 0.75)|}{-(|x| - 0.5)(|x| - 0.75)}}$$

$$\text{Forehead : } p(x) = 2.25\sqrt{\frac{|1 - (x - 0.5)(x + 0.5)|}{-(x - 0.5)(x + 0.5)}}$$

$$\text{Shoulders : } q(x) = \frac{6\sqrt{10}}{7} + (1.5 - 0.5|x|)\sqrt{\frac{|x| - 1}{|x| - 1}} - \frac{6\sqrt{10}}{14}\sqrt{4 - (|x| - 1)^2}$$

$$\text{Wings : } r(x) = 3\sqrt{1 - \left(\frac{x}{7}\right)^2}\sqrt{\frac{|x| - 3}{|x| - 3}} \quad s(x) = -3\sqrt{1 - \left(\frac{x}{7}\right)^2}\sqrt{\frac{|x| - 4}{|x| - 4}}$$

Lesson 1

A-Level Pure Mathematics : Year 2 Integration III

1.1 Integration by Substitution

Example

A-Level Examination Question from January 2006, Paper C4, Q3 (Edexcel)

Using the substitution $u^2 = 2x - 1$, or otherwise, find the exact value of

$$\int_1^5 \frac{3x}{\sqrt{2x-1}} dx$$

[8 marks]

1.2 Exercise

*Any solution based entirely on graphical
or numerical methods is not acceptable*

Marks Available : 50

Question 1

Evaluate the following integral using the substitution given;

$$\int_3^{11} \frac{x}{\sqrt{x-2}} dx \qquad u^2 = x - 2$$

[8 marks]

Question 2

Evaluate the following integral using the substitution given;

$$\int_6^{10} \frac{x(x-4)}{(x-2)^2} dx \qquad u = x - 2$$

[8 marks]

Question 3

Evaluate exactly the following integral using the substitution given;

$$\int_2^6 \frac{x-1}{\sqrt{2x-3}} dx \qquad u^2 = 2x-3$$

[8 marks]

Question 4

Evaluate the following integral using the substitution given;

$$\int_{0.5}^1 \frac{1}{\sqrt{1-x^2}} dx \quad x = \sin \theta$$

Warning : You must use **RADIANS**

[8 marks]

Question 5

Evaluate the following integral using the substitution given;

$$\int_2^{2.5} \frac{2x + 1}{(3 - x)^6} dx \qquad u = 3 - x$$

[9 marks]

Question 6

Evaluate the following integral using the substitution given;

$$\int_{-0.25}^{0.5} \frac{48x}{(2x+1)^5} dx \quad u = 2x + 1$$

[9 marks]

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Teachers may obtain detailed worked solutions to the exercises by email from MHHShrewsbury@Gmail.com